

13. Gravitational fields

Define gravitational potential at a point

- Work done per unit mass
- In moving the mass from infinity to that point

A satellite X, of mass M , orbits a planet at a constant distance $4R$ from the centre of the planet, as shown in Fig. 1.1.

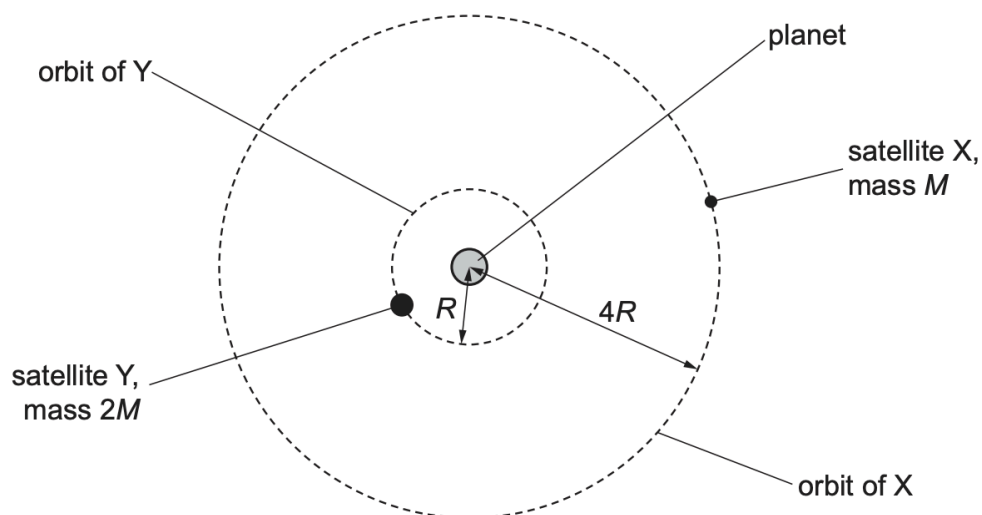


Fig. 1.1 (not to scale)

A second satellite Y, of mass $2M$, orbits the planet with orbital radius R .

The gravitational potential at X due to the planet is $-\Phi$. The planet is a uniform sphere.

a. Explain why the gravitational potential at X is negative.

- Gravitational potential at infinity is zero.
- Work is done by (two) masses in moving them closer together OR work is done on (two) masses in moving them apart.

b. State an expression, in terms of Φ , for the gravitational potential at Y due to the planet.

- -4Φ

NOTE: gravitational potential is only dependent on the mass of the object creating the gravitational field.

c.

	Satellite X	Satellite Y
Gravitational field	$\Phi / 4R$	$4\Phi / R$

strength at satellite due to planet		
Gravitational potential energy of satellite	$-M\phi$	$-8M\phi$

State what is indicated by the direction of the gravitational field line at a point in a gravitational field.

- Direction of the force acting on a test mass placed at that point.

Explain, with reference to gravitational field lines, why the gravitational field near the surface of the Earth is approximately constant for small changes in height.

- change in height negligible compared with radius of Earth.
- so field lines are effectively parallel.

A large isolated uniform sphere has mass M and radius R .

Point P lies on a straight line passing through the centre of the sphere, at a variable displacement x from the centre, as shown in Fig. 1.1.

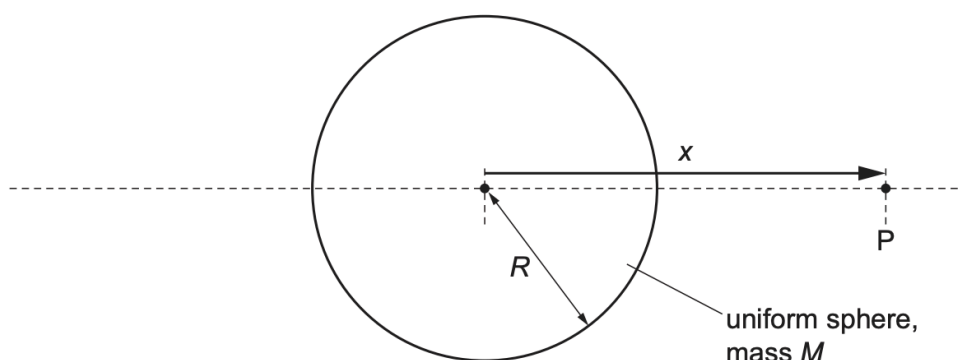


Fig. 1.1

a. Determine an expression for Y in terms of M and R . Identify any other symbols that you use.

- $Y = GM / R^2$
- G is the gravitational constant

b. Explain why, at the surface of the sphere, g always has the opposite sign to x .

- gravitational force is (always) attractive OR gravitational force (always) acts towards the centre of the sphere
- force is in opposite direction to displacement OR at a point to the right of the centre, force acts to the left OR at a point to the left of the centre, force acts to the right.

c.

Fig. 1.2 shows the variation with x of the gravitational field g at point P due to the sphere for the values of x for which P is inside the sphere.

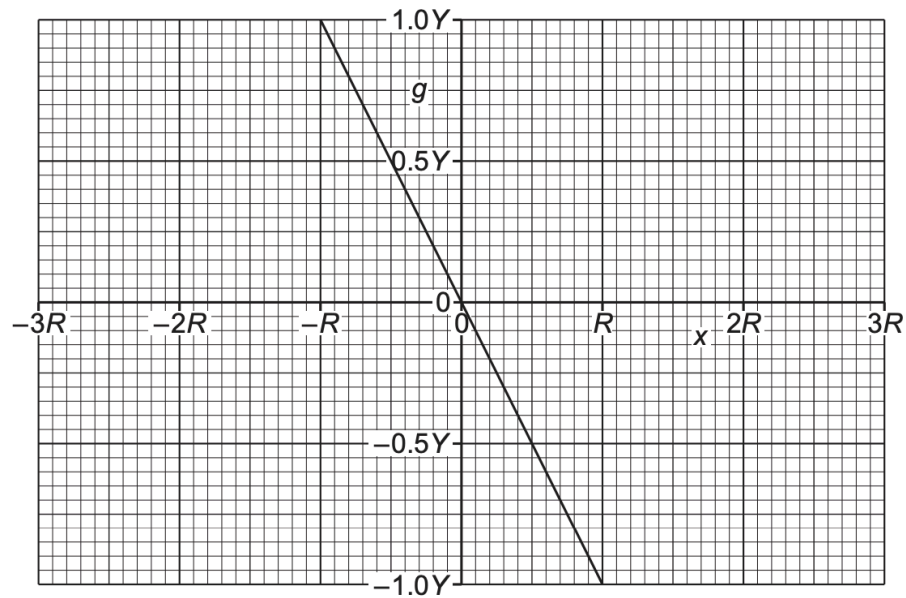
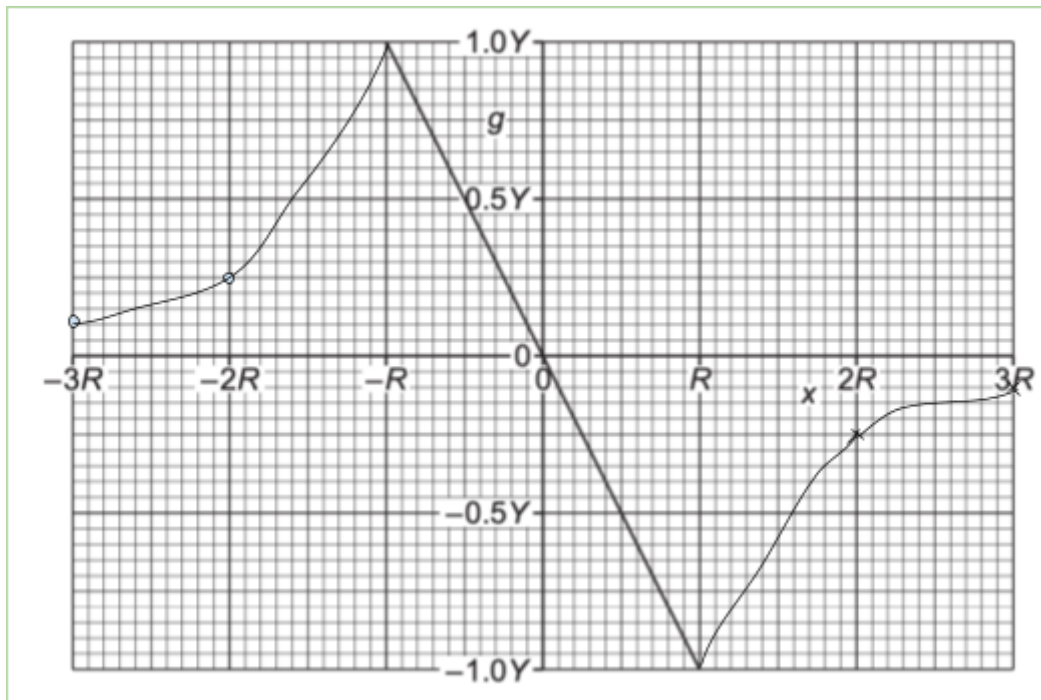


Fig. 1.2

Complete Fig. 1.2 to show the variation of g with x for values of x , up to $\pm 3R$, for which point P is outside the sphere.



sketch: smooth curve with decreasing positive gradient, starting at $(R, -Y)$ and reaching $3R$ with g still negative
or
 smooth curve with increasing positive gradient, ending at $(-R, Y)$ and reaching $-3R$ with g still positive

both of the above curves, in correct quadrants

curve passing through $(\pm 2R, \pm 0.25Y)$ and $(\pm 3R, \pm 0.11Y)$

- At $2R$, g will become $\frac{1}{4}$ and at $3R$, g will become $\frac{1}{9}$. Plot these values.

State Newton's law of gravitation.

- Gravitational force is directly proportional to the product of the masses
- And inversely proportional to the square of their separation.

Explain why there is a point X on the line between the centres of the Earth and the Moon where the resultant gravitational field strength due to the Earth and the Moon is zero.

- Fields due to earth and moon have equal magnitude
- But act in opposite directions

The Earth has a mass of $5.98 \times 10^{24} \text{ kg}$ and a radius of $6.37 \times 10^6 \text{ m}$.

The Moon has a mass of $7.35 \times 10^{22} \text{ kg}$ and a radius of $1.74 \times 10^6 \text{ m}$.

The Earth and the Moon can both be considered as point masses at their centres. Their centres are a distance of $3.84 \times 10^8 \text{ m}$ apart.

There is a point X on the line between the centres of the Earth and the Moon where the resultant gravitational field strength due to the Earth and the Moon is zero.

Calculate the distance x of point X from the centre of the Moon.

distance of X from Earth = $(3.84 \times 10^8 - x)$
$(G \times) 7.35 \times 10^{22} / x^2 = (G \times) 5.98 \times 10^{24} / (3.84 \times 10^8 - x)^2$
$x = 3.8 \times 10^7 \text{ m}$

14. Temperature

Explain why temperature measured using a laboratory liquid-in-glass thermometer does not give a measurement of thermodynamic temperature.

- Measurement depends on the properties of the liquid

(b) Fig. 2.1 shows a simplified diagram of a type of thermometer called a platinum resistance thermometer.

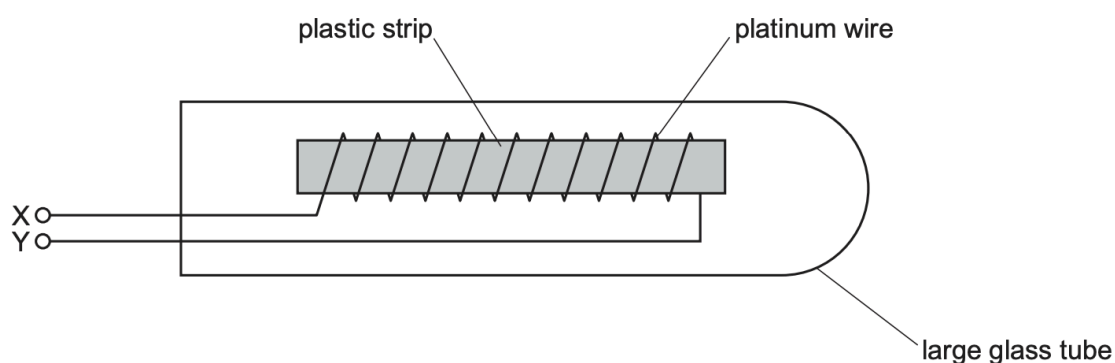
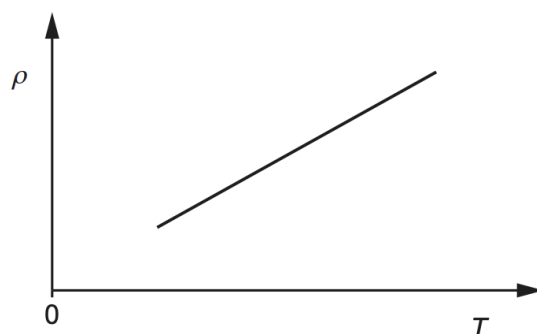


Fig. 2.1

The glass tube is immersed in the environment for which the temperature is to be determined. The resistance between the terminals X and Y is measured.

Fig. 2.2 shows the variation of the resistivity ρ of platinum with thermodynamic temperature T .



a. Explain how Fig. 2.2 shows that platinum is a suitable metal for use in a resistance thermometer.

- resistivity varies with temperature
- variation with temperature is linear
- unique value of resistivity for each different value of temperature

b. Suggest a reason why a platinum resistance thermometer is not suitable for measuring a rapidly changing temperature.

- thermometer has high heat capacity/specific heat capacity OR energy transfer needed for thermometer to reach correct temperature OR thermometer takes time to reach the correct temperature

c. A negative temperature coefficient thermistor may be used as a type of resistance thermometer. State one way in which the variation with temperature of the resistance of a thermistor differs from that of a platinum wire.

- variation is inverse OR variation is non-linear

Suggest a type of thermometer that is suitable for measuring a rapidly changing temperature.

- Thermocouple

Define specific heat capacity.

- thermal energy per unit mass to change temperature
- thermal energy per unit change in temperature

NOTE:

when asked to state the unit of specific heat capacity, always give it in $\text{J kg}^{-1} \text{K}^{-1}$

15. Ideal gases

State what is meant by an ideal gas.

- gas for which $pV \propto T$
- where p is pressure, V is volume, and T is thermodynamic temperature

Use one of the basic assumptions of the kinetic theory to explain what can be deduced about the potential energy associated with the random motion of molecules in an ideal gas.

- no intermolecular forces
- so potential energy is zero

NOTE:

- Internal energy = total KE + PE of molecules
- For ideal gas, PE = 0
- Internal energy = total KE = $\frac{3}{2} NkT$

NOTE:

$$pV = \frac{1}{3} Nm \langle c^2 \rangle$$

- N: number of molecules (of the gas)
- m: mass of one molecule (of the gas)
- $\langle c^2 \rangle$: mean square speed (of molecules)

Gas at the surface of a star has a very high pressure. Use the basic assumptions of the kinetic theory to suggest why, in practice, a gas at the surface of a star is unlikely to behave as an ideal gas.

- very high pressure so molecules are very close together
- forces between molecules are not negligible OR volume of molecules not negligible compared with gas volume

16. Thermodynamics

An ideal gas of mass 0.35 kg is heated at a constant pressure of $2.0 \times 10^5 \text{ Pa}$ so that its internal energy increases by 7600 J. During this process, the volume of the gas increases from 0.038 m^3 to 0.063 m^3 and the temperature increases by 56°C .

a. Explain whether the work done on the gas is positive or negative.

- gas is expanding against external pressure
- gas does work / work is done by gas, so work done on gas is negative

b. Determine the magnitude of the thermal energy q transferred to the gas.

$\Delta U = q + W$
$7600 = q + (-5000)$
$q = 12600 \text{ J}$

c. Calculate the specific heat capacity of the gas for this process. Give a unit with your answer.

specific heat capacity = $q / m \Delta T$
$= 12600 / (0.35 \times 56)$
$= 640 \text{ J kg}^{-1} \text{ K}^{-1}$

d. The gas is now heated at constant volume rather than at constant pressure. The increase in internal energy of the gas is the same. Use the first law of

thermodynamics to explain whether the specific heat capacity of the gas for this process is less than, the same as, or greater than the answer in c.

- Same gain in internal energy so same temperature rise.
- no change in volume so no work done OR no work done so less thermal energy needed for same change in internal energy.
- less thermal energy needed for same temperature change, so lower specific heat capacity.

A fixed mass of an ideal gas is initially at a temperature of 17°C .
The gas has a volume of 0.24 m^3 and a pressure of $1.2 \times 10^5\text{ Pa}$.

(b) The gas undergoes three successive changes, as shown in Fig. 3.1.

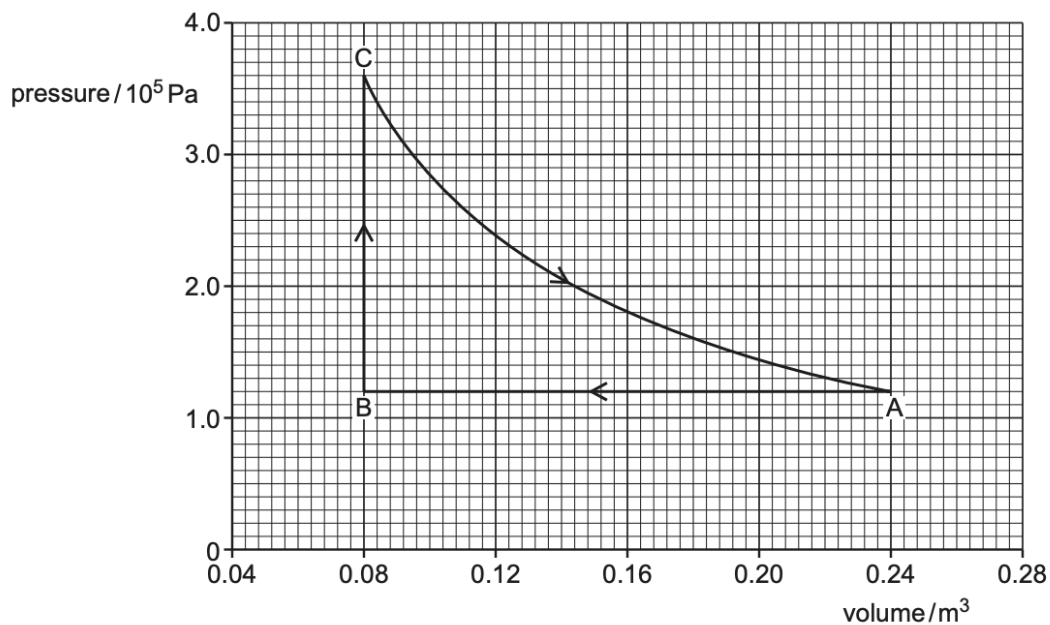


Fig. 3.1

The initial state is represented by point A. The gas is cooled at constant pressure to point B by the removal of 48.0 kJ of thermal energy.

The gas is then heated at constant volume to point C.

Finally, the gas expands at constant temperature back to its original pressure and volume at point A. During this expansion, the gas does 31.6 kJ of work.

- (ii) Complete Table 3.1 to show the work done on the gas, the thermal energy supplied to the gas and the increase in internal energy of the gas, for each of the changes AB, BC and CA.

Table 3.1

change	work done on gas/kJ	thermal energy supplied to gas/kJ	increase in internal energy of gas/kJ
AB	19.2	-48.0	-28.8
BC	0	+28.8	+28.8
CA	-31.6	+31.6	0

[5]

- Work done in AB = $p\Delta V = 1.2 \times 10^5 \times (0.24 - 0.08) = 19.2 \text{ kJ}$
- Work done in BC = 0, because there is no change in volume
- Increase in internal energy in CA = 0, because no increase in temperature
- The internal energies should add up to give 0, because the gas returns to its original pressure and volume.

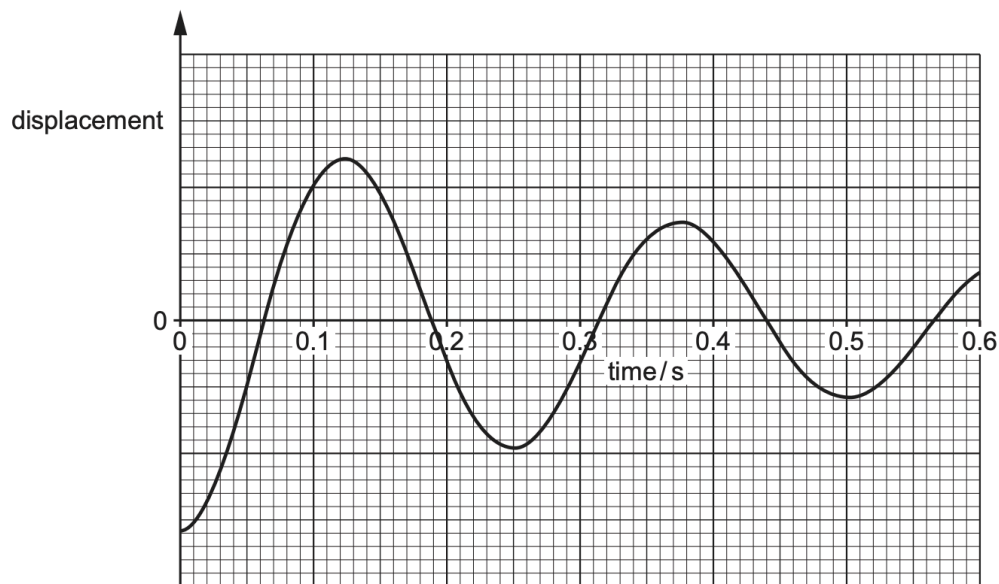
17. Oscillations

State what is meant by resonance.

- oscillation of object at maximum amplitude
- when driving frequency = natural frequency of system

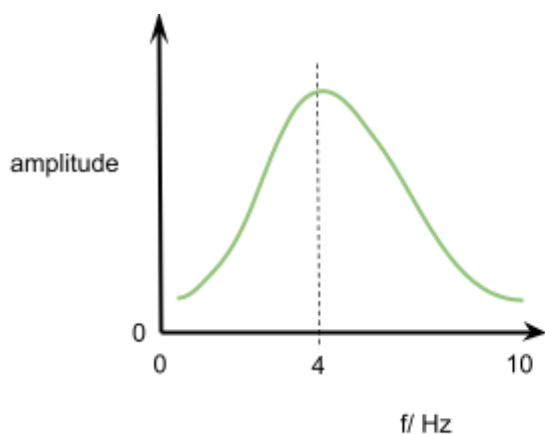
Explain the decrease with time of the amplitude of the oscillations of a lightly damped oscillator.

- oscillations lose energy
- due to resistive forces acting on oscillator



Frequency of oscillations = 4.0Hz

Vibration generator is switched on and its frequency f of vibration is gradually increased from 0 to 10Hz. Sketch the variation with f of the amplitude of the oscillations of the ball.



- Natural frequency = 4Hz.
- Maximum amplitude when driving frequency = natural frequency.

State what is meant by simple harmonic motion.

- (motion in which) acceleration is directly proportional to displacement.
- (motion in which) acceleration is (always) in the opposite direction to displacement OR acceleration is (always) directed towards a fixed point.

State how a–x graph shows that the motion of an oscillator is simple harmonic.

- Straight line passing through origin shows that a is directly proportional to x .
- Negative gradient shows that a acts in opposite direction to x .

A pendulum consists of a bob (small metal sphere) attached to the end of a piece of string. The other end of the string is attached to a fixed point. The bob oscillates with small oscillations about its equilibrium position, as shown in Fig. 4.1.

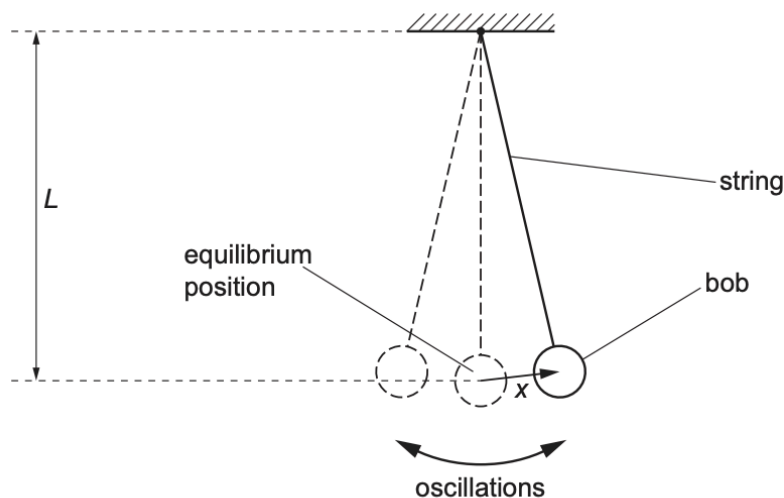


Fig. 4.1 (not to scale)

The length L of the pendulum, measured from the fixed point to the centre of the bob, is 1.24 m.

The angular frequency ω is related to the length L of the pendulum by

$$\omega = \sqrt{\frac{k}{L}}$$

where k is a constant.

$$\omega = 2.8 \text{ rad s}^{-1}$$

a. Determine k . Give a unit with your answer.

$k = \omega^2 L$
$= 2.8^2 \times 1.24$
$= 9.7 \text{ m s}^{-2}$

NOTE: using the same units on both sides method, you get the unit of k as $\text{rad}^2 \text{ s}^{-2} \text{ m}$. Since rad is just a unit for small angle, ignore it and unit becomes m s^{-2} .

b. While the pendulum is oscillating, the length of the string is increased in such a way that the total energy of the oscillations remains constant. Suggest and explain the qualitative effect of this change on the amplitude of the oscillations.

- Increasing L causes ω to decrease OR
 $\text{energy} (= \frac{1}{2} m \omega^2 x_0^2) = \frac{1}{2} m k x_0^2 / L$ (and L increases)

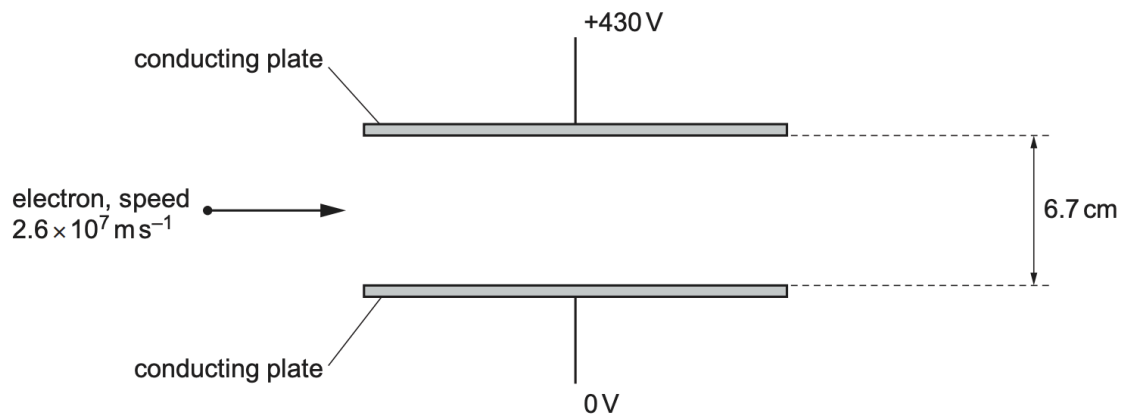
- So amplitude increases

18. Electric fields

Define electric field.

- force per unit charge
- force on positive charge

(b) Fig. 5.1 shows two parallel conducting plates that are in a vacuum. The plates are separated by a distance of 6.7 cm and have a potential difference (p.d.) of 430 V between them.



a. Draw the path of an electron as it moves between and beyond the plates.

- smooth curve within plates and straight lines outside plates.
- direction of deflection shown as upwards.

b. A uniform magnetic field is now applied in the region of the electric field, so that the electron travels undeviated. Explain, with reference to the forces exerted by the two fields, why the path of the electron is undeviated.

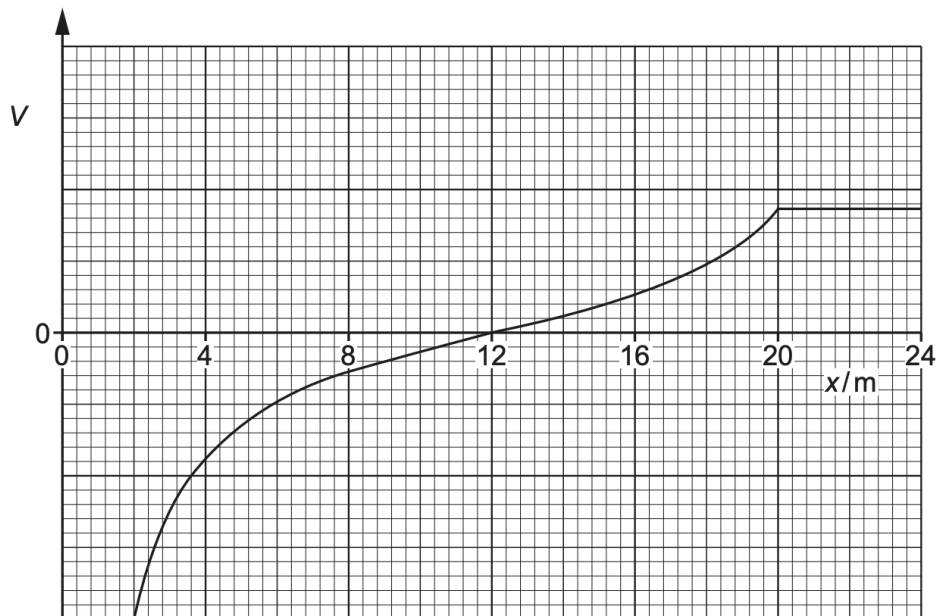
- forces are in opposite directions
- undeviated when magnitudes of forces are equal

Define electric potential at a point.

- Work done per unit charge.
- Work done on charge in moving positive charge from infinity to the point.

Two isolated charged metal spheres X and Y are situated near to each other in a vacuum with their centres a distance of 24 m apart. Point P is at a variable distance x from the centre of sphere X on the line joining the centres of the spheres.

Fig. 5.1 shows the variation with x of the electric potential V due to the spheres at point P.



a. State three conclusions that can be drawn about the spheres from Fig. 5.1. The conclusions may be qualitative or quantitative.

- radius of sphere X is 2.0 m, radius of sphere Y is 4.0 m, radius of Y is double the radius of X.
- charge on X is negative, charge on Y is positive, spheres carry opposite charges.
- magnitudes of charges on the spheres are equal. (use equation $V = Q / 4\pi\epsilon_0 r$)

b. A positively charged particle is placed at point P such that $x = 12$ m. The particle is released. Describe and explain the subsequent motion of the particle.

- particle is attracted to X or repelled from Y OR resultant force on particle is towards X / away from Y / to the left.
- particle accelerates towards X / away from Y / to the left.
- magnitude of acceleration of particle increases.

19. Capacitance

Fig. 6.1 shows a capacitor of capacitance C connected in series with a resistor of resistance R .

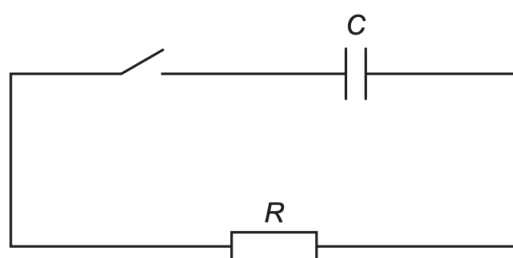


Fig. 6.1

Initially the switch is open and there is a p.d. of 12V across the capacitor.

At time $t = 0$, the switch is closed so that there is a current I in the resistor.

Fig. 6.2 shows the variation of I with t .

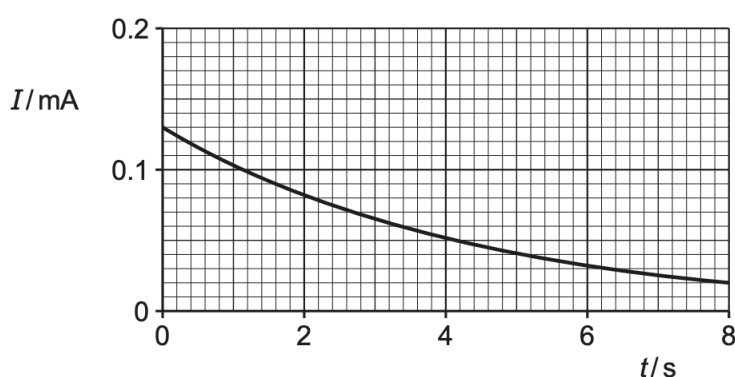


Fig. 6.2

Explain the shape of the line in Fig. 6.2.

- p.d. across capacitor proportional to charge on capacitor
- p.d. across capacitor = p.d. across resistor
- current in resistor proportional to p.d. across resistor
- current in resistor = rate of decrease of charge on capacitor
- charge proportional to current so current decreases as rate of decrease of charge decreases, therefore exponential shape

NOTE: in a circuit for discharging capacitor: resistor, capacitor and switch are all in series. The capacitor acts as the power supply, since there is no other power supply. Thus p.d. across the capacitor = p.d. across the resistor.

20. Magnetic fields

Define magnetic flux density.

- force per unit length
- force per unit current
- length / current perpendicular to field

A sphere of mass $1.6 \times 10^{-10} \text{ kg}$ has a charge of $+0.27 \text{ nC}$. The sphere is in a uniform electric field that acts vertically upwards, as shown in the side view in Fig. 2.1.

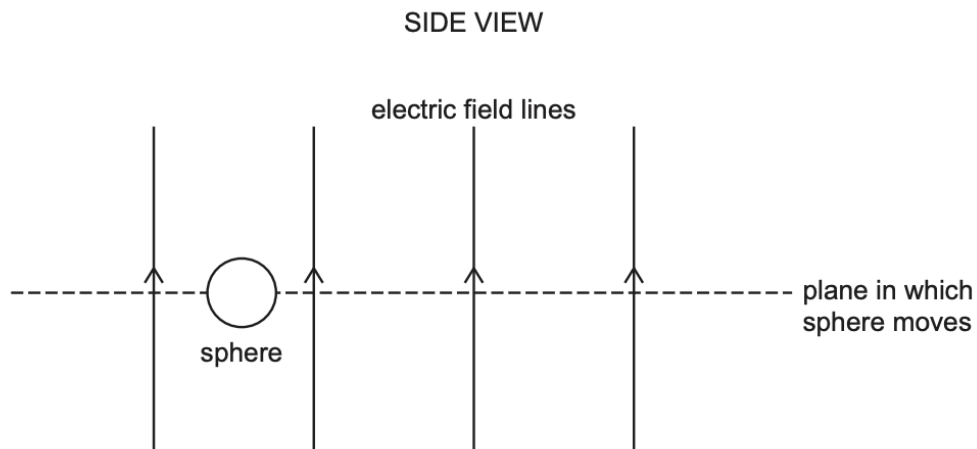


Fig. 2.1

The force exerted on the sphere by the electric field causes the sphere to remain at a fixed vertical height in a horizontal plane.

There is a uniform magnetic field in the region of the electric field. The sphere moves at a speed of 0.78 m s^{-1} in the horizontal plane. The magnetic field causes the sphere to move in a circular path of radius 3.4 m , as shown in the view from above in Fig. 2.2.

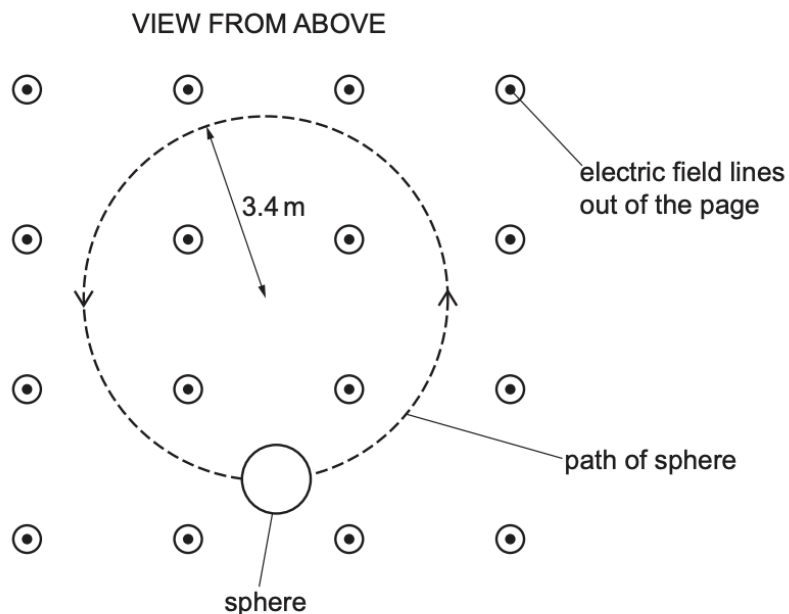


Fig. 2.2

a. Determine the direction of the uniform magnetic field.

- Vertically downwards

Explaining this entire figure

- Electric field is vertically upwards, so electric force acts vertically upwards.
- The electric force alone causes the sphere to remain at a fixed vertical height, so this means that the electric force would be equal to weight. $F_E = mg$
- Now magnetic field causes the sphere to move in a circular path.
- Which means, magnetic force = centripetal force that acts towards the centre of the circle at all times.
- When figuring out direction of magnetic field, ignore the electric field lines.
- Use LH rule- taking middle finger as velocity of sphere (current), thumb as magnetic force (which is perpendicular to velocity), and the index shows direction of field as into the page.
- When you look back at fig. 2.1, into the page corresponds to vertically downwards from the side view.

b. Explain why the motion of the sphere in the horizontal plane is circular.

- magnetic force (on sphere) is perpendicular to its velocity
- magnetic force perpendicular to velocity is the centripetal force OR magnetic force perpendicular to velocity causes centripetal acceleration OR acceleration perpendicular to velocity is centripetal (acceleration) OR magnetic force does not change the speed of the sphere OR magnetic force has constant magnitude

c. Calculate the strength of the uniform electric field.

$mg = Eq$
$E = (1.6 \times 10^{-10} \times 9.81) / (0.27 \times 10^{-9})$
$= 5.8 \text{ N C}^{-1}$

d. By considering the magnetic force on the sphere, show that the flux density of the uniform magnetic field is 0.14T.

centripetal force = magnetic force or $Bqv = mv^2 / r$
$B = mv / qr$
$= (1.6 \times 10^{-10} \times 0.78) / (0.27 \times 10^{-9} \times 3.4) = 0.14 \text{ T}$

Define magnetic flux.

- product of magnetic flux density and area
- where area is perpendicular to the magnetic field

NOTE: unit of magnetic flux linkage is just Wb.

Induced emf = rate of change of flux linkage!!

A square coil of wire of side length 12 cm consists of 8 insulated turns. The coil is stationary in a uniform magnetic field. The plane of the coil is perpendicular to the magnetic field, as shown in Fig. 6.1.

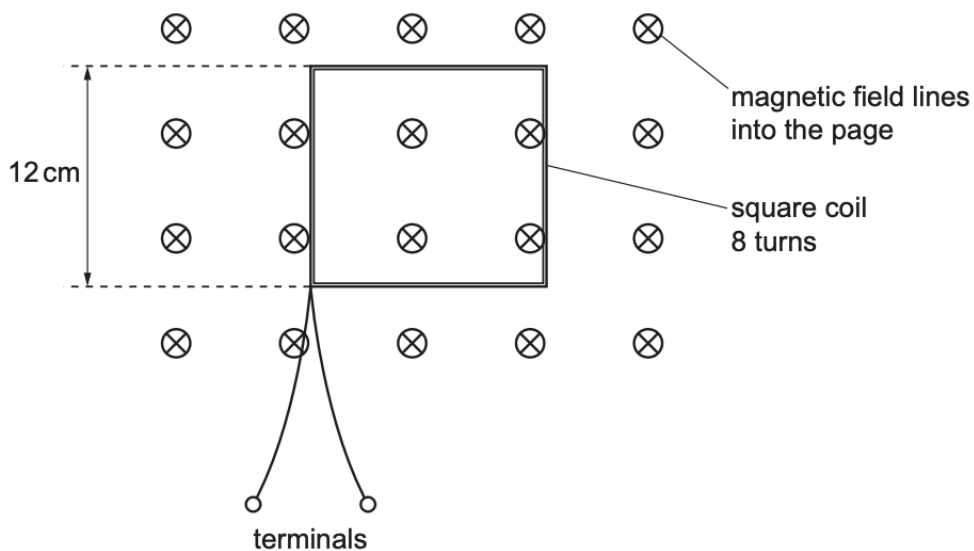


Fig. 6.1

The flux density B of the magnetic field varies with time t as shown in Fig. 6.2.

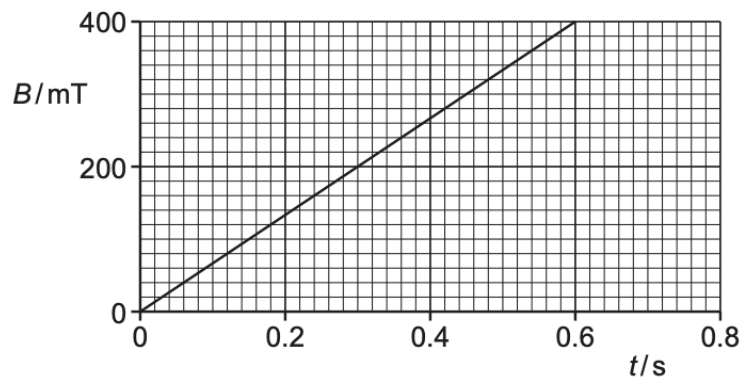


Fig. 6.2

- State how Fig. 6.2 shows that the electromotive force (e.m.f.) E induced across the terminals between $t = 0$ and $t = 0.60s$ is constant.
 - Straight line
- The procedure in is repeated, but this time the terminals of the coil are connected together. State and explain the effect on the coil of connecting the

terminals together during the change of magnetic flux density shown in Fig. 6.2.

- induced e.m.f. causes current flow in the coil (because the circuit is now complete)
- either
- current in magnetic field causes forces to act on the coil
- opposite sides of coil forced inwards
- or
- current causes dissipation of energy in the resistance of the coil
- temperature of the coil rises

21. Alternating currents

A varying current I passes through a resistor of resistance R in the circuit shown in Fig. 7.1.

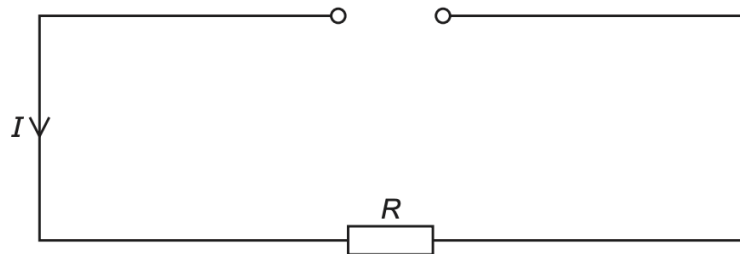


Fig. 7.1

Fig. 7.2 shows the variation with time t of I .

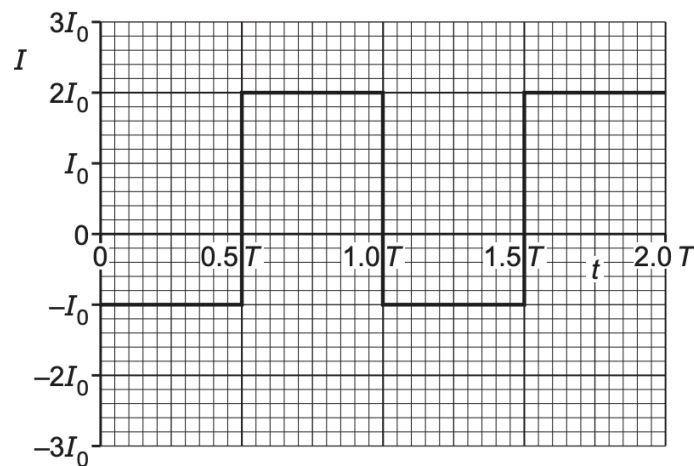
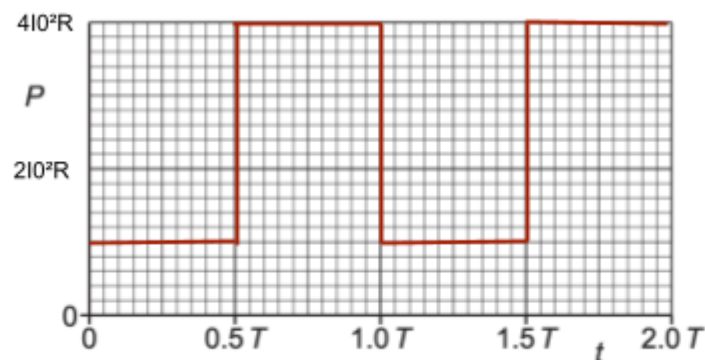


Fig. 7.2

The current has magnitude $2I_0$ when it is in the positive direction and I_0 when it is in the negative direction. The period of the variation of the current is T .

a.

On Fig. 7.3, sketch the variation of P with t between $t = 0$ and $t = 2.0T$. Label the power axis with an appropriate scale.



b. Use your answer in (a) to determine an expression, in terms of I_0 and R , for the mean power $\langle P \rangle$ in the resistor.

$$- \quad 5/2 I_0^2 R$$

c. Use your answer in (a) to determine an expression, in terms of I_0 and R , for the root-mean-square current $I_{\text{r.m.s}}$ in the resistor.

$\langle P \rangle = I_{\text{r.m.s.}}^2 R$
$I_{\text{r.m.s.}}^2 R = (5/2) I_0^2 R$
$I_{\text{r.m.s.}} = \sqrt{(5/2)} I_0$

State what is meant by peak voltage

- The maximum voltage of an alternating voltage.

State what is meant by root-mean-square voltage

- The value of steady/ constant voltage
- That produces same power in a resistor as the alternating voltage

OR

- If the alternating voltage is squared and averaged
- The rms value is the square root of this average value

State the meaning of rectification

- Process of converting alternating current into direct current.

State the meaning of smoothing

- The reduction in the variation of the output current/voltage.

A capacitor is placed in parallel with a resistive load to smooth the rectified voltage. The graph of the smoothed output voltage against time gives a 'ripple' shape as shown in Fig. 1.2.



State how the ripples in the graph can be reduced

- A capacitor with greater capacitance
- A resistor with larger resistance

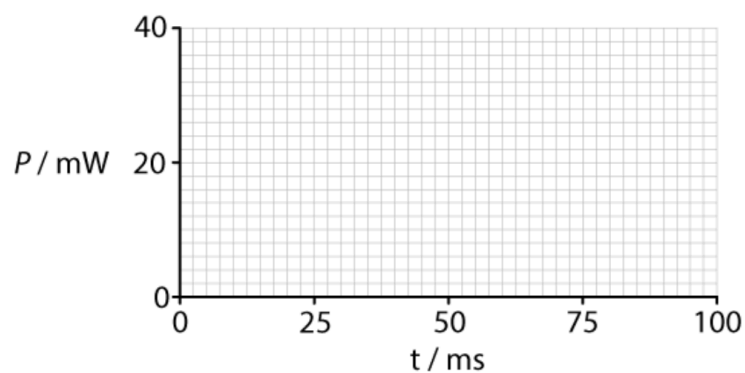
A sinusoidal alternating voltage has a root-mean-square voltage of 5.1 V and a frequency of 40 Hz .

The alternating voltage is applied across a resistor of resistance $1430\ \Omega$.

Calculate the mean power dissipated by the resistor in mW.

On Fig. 1.1, draw a smooth curve to show how the power P dissipated in the resistor varies with time t between $t = 0$ and $t = 100\text{ ms}$.

Assume that $P = 0$ when $t = 0$.



Ans:

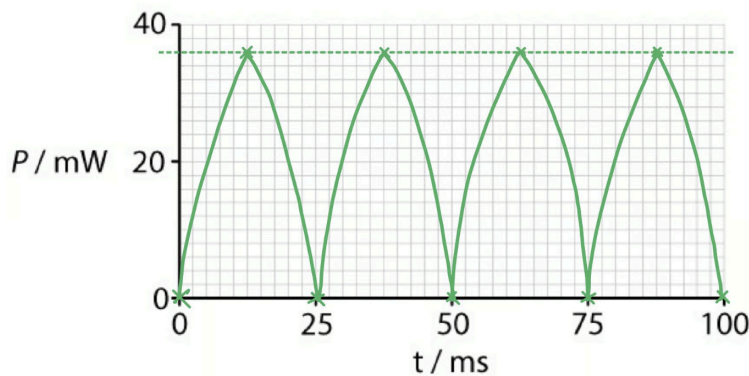
- Mean power, $P_{mean} = 18 \text{ mW}$ (from part a)
- Frequency of wave, $f = 40 \text{ Hz}$ (from part a)

Calculate the peak power:

- $P_{peak} = 2 \times P_{mean}$
- $P_{peak} = 2 \times 18$
- $P_{peak} = 36 \text{ mW}$ [1 mark]

Calculate the time period of the wave, T :

- $T = \frac{1}{f} = \frac{1}{40}$
- $T = 0.025 \text{ s} = 25 \text{ ms}$



An alternating voltage with frequency 40Hz is applied to a piezoelectric crystal in air. State and explain:

- **The effect this has on the air surrounding the crystal:**
 - Alternating voltage causes the crystal to vibrate
 - Vibrations of the crystal cause the air to vibrate
- **Which quantity must change if the crystal were to be used in an ultrasound transducer?**
 - The frequency of the vibrations must be increased
 - Frequency of 40Hz is too low for ultrasound

The output voltage V of an a.c. power supply varies sinusoidally with time t as shown in Fig. 1.1.

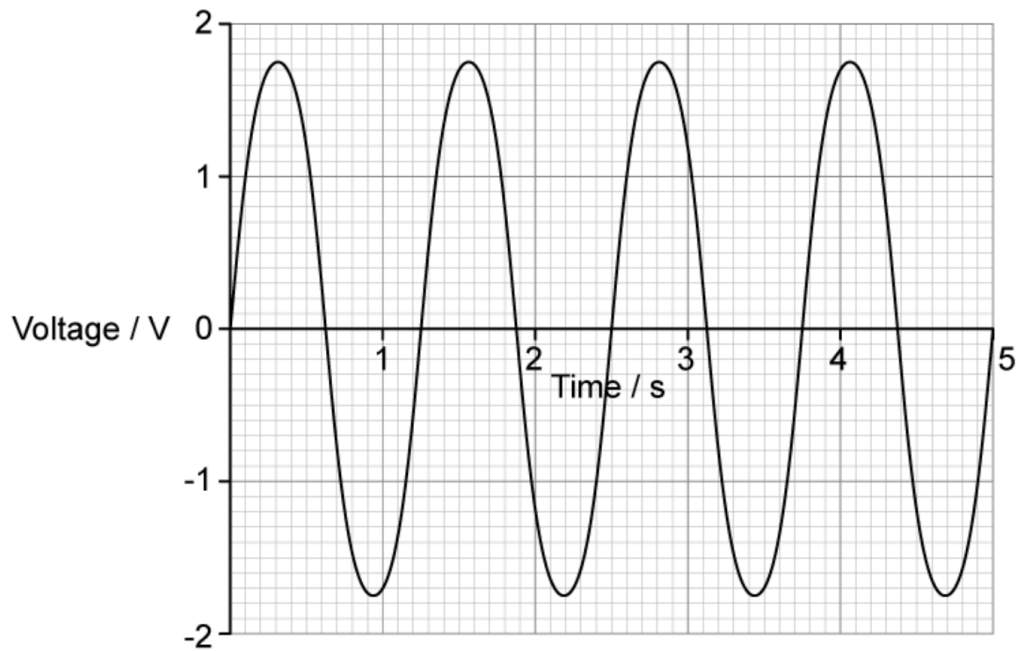


Fig. 1.1

Using Fig. 1.1, determine the equation for V in terms of t , where V is in volts and t is in seconds.

Ans:

- From the graph: time period, $T = 1.25$ s
AND
- From the graph: peak voltage, $V_0 = 1.75$ V [1 mark]

Calculate the angular frequency, ω :

- Angular frequency: $\omega = \frac{2\pi}{T}$
- Angular frequency: $\omega = \frac{2\pi}{1.25} = \frac{8\pi}{5}$ (5.03 rad s^{-1}) [1 mark]

Use the sinusoidal representation for voltage:

- Alternating voltage: $V = V_0 \sin \omega t$
- Alternating voltage: $V = 1.75 \sin \left(\frac{8\pi}{5} t \right)$ or $V = 1.75 \sin (5.03t)$ [1 mark]

Using Fig. 1.1, determine

(i) the root-mean-square voltage V_{rms}

[1]

(ii) the mean voltage $\langle V \rangle$

[1]

Ans:

(i) Determine the root-mean-square voltage:

- Peak voltage, $V_0 = 1.75 \text{ V}$ (from part a)
- Root-mean-square voltage: $V_{rms} = \frac{V_0}{\sqrt{2}}$
- Root-mean-square voltage: $V_{rms} = \frac{1.75}{\sqrt{2}} = 1.24 \text{ V}$ [1 mark]

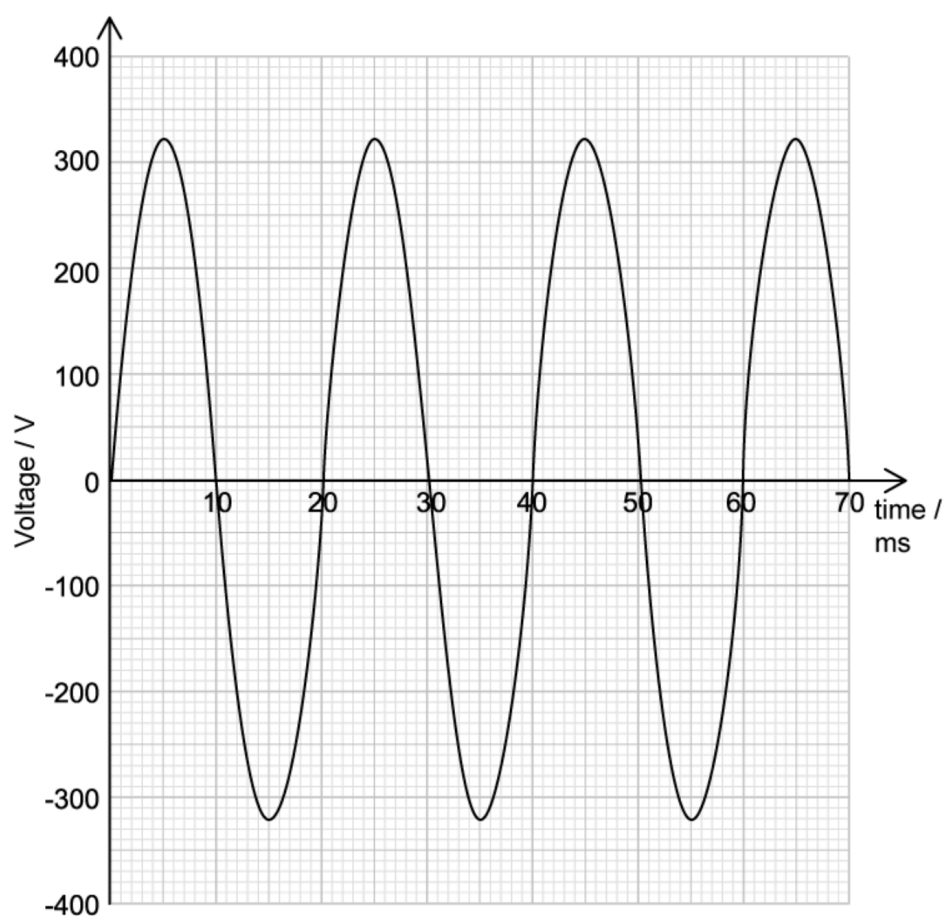
(ii) Determine the mean voltage:

- Mean voltage = 0 V [1 mark]

Explain, by referring to the heating effect, what is meant by the r.m.s value of an alternating current.

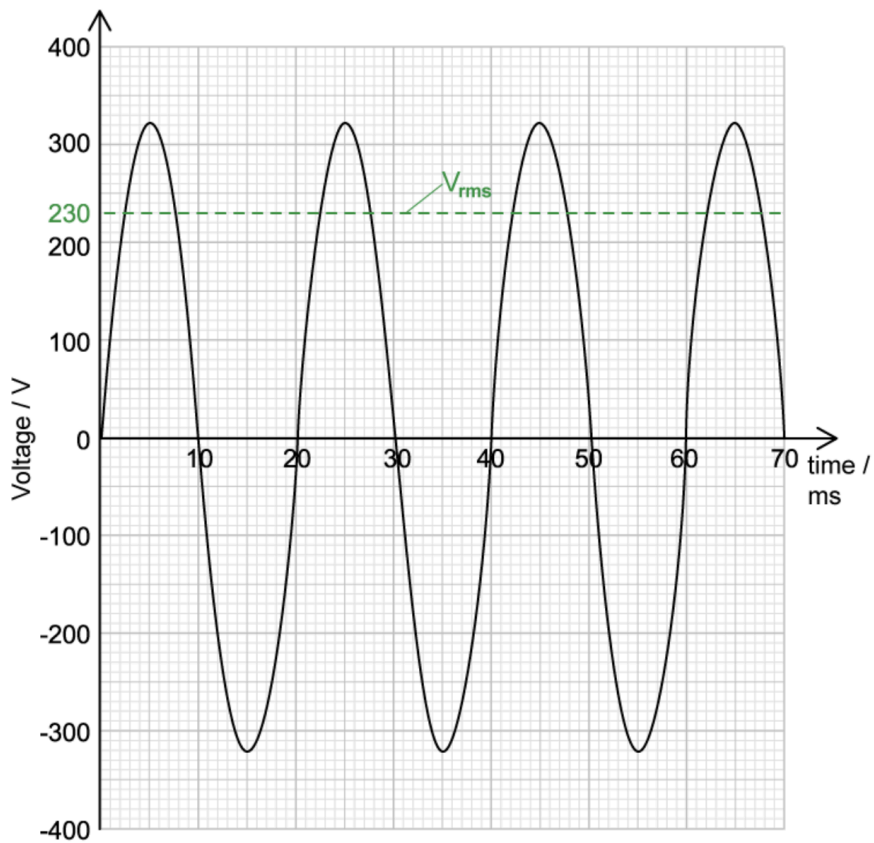
- The direct/ steady current
- That dissipates heat/ thermal energy at the same rate in a resistor OR that produces the same power dissipation/ heating effect in a resistor.

Fig. 1.1 below shows the graph of the voltage against time for a mains supply in the UK.



Using Fig. 1.1, draw a line to show the dc voltage that gives the same power as produced by the alternating waveform in the mains supply.

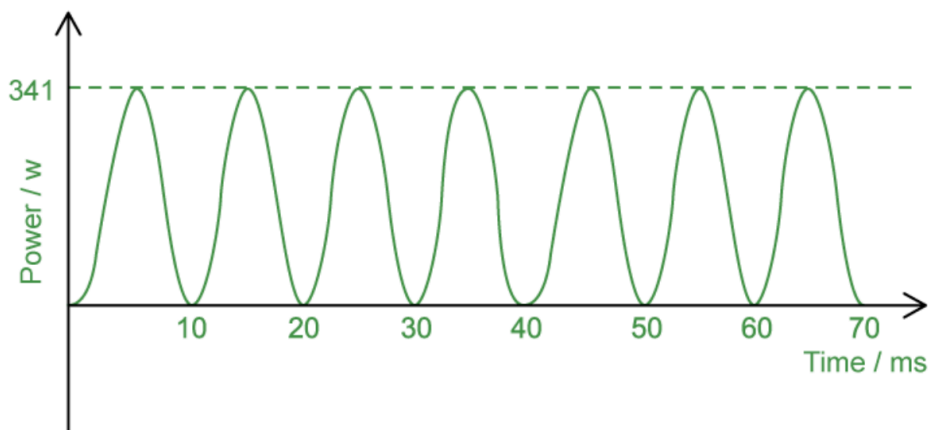
Ans:



Sketch a graph which shows how the power supplied by this voltage to a resistor with resistance of $300\ \Omega$ varies with time.

Label the vertical axis as power and mark on both axes any significant values.

Ans:



The signal in Fig. 1.1 is connected to an oscilloscope. The screen's grid has a height of 8 squares and a width of 10 squares.

Describe how you would use the oscilloscope to display the alternating waveform in Fig. 1.1 so that four complete cycles are visible and occupy the full height of the screen.

Ans:

- Adjust the time-base to 8ms div^{-1}
- Adjust the y-gain to 80V div^{-1}

An electric oven is connected to the mains supply from part (a), using a cable with a non-negligible resistance. The cable connects the heating element in the oven to the mains supply.

Explain why the rms voltage across the heating element in the oven will be less than the rms voltage calculated in part (a).

The rms voltage across the heating element will be less than 230 V because:

- There is a potential difference / voltage across the cable; [1 mark]
- The (rms) voltage across the oven is 230 V / the mains minus the voltage across the cable; [1 mark]

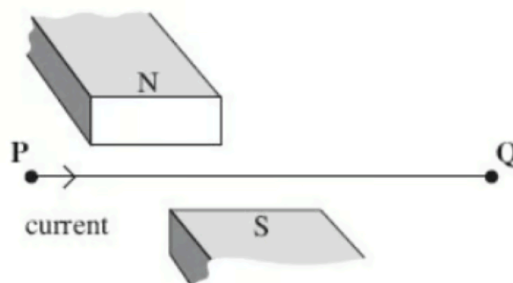
OR

- The resistance of the cable lowers the current in the circuit; [1 mark]
- The voltage across the oven is lower because of $V = IR$ / Ohm's law; [1 mark]

OR

- The resistance of the cable is in series with the heating element; [1 mark]
- The voltage splits in ratio across these resistances; [1 mark]

Fig. 1.1 below shows a horizontal wire, held in tension between fixed points at P and Q. A short section of the wire is positioned between the pole pieces of a permanent magnet, which applies a uniform horizontal magnetic field at right angles to the wire. Wires connected to a circuit at P and Q allow an alternating current to be passed through the wire.



Explain why the wire oscillates

- The wire has a magnetic field
- The wire's magnetic field interacts with the uniform magnetic field

- The current flowing through the wire alternates
- The magnetic field around the wire also alternates
- The force on the wire alternates direction

(ii) The permanent magnet produces a uniform magnetic field of flux density 220 mT over a 55 mm length of the wire. Show that the maximum force on the wire is about 40 mN when there is an alternating current of rms value 2.4 A in it.

Ans:

Determine the peak current, I_0 :

- $I_{rms} = \frac{I_0}{\sqrt{2}}$
- $I_0 = 2.4 \times \sqrt{2} = 3.39 \text{ A}$ [1 mark]

Calculate the maximum force:

- Maximum force occurs at peak current
- $F_{max} = BI_0L = 0.220 \times 3.39 \times 0.055$
- $F_{max} = 0.041 \text{ N} = 41 \text{ mN}$ [1 mark]
- This is roughly 40 mN

When modelling the motion of the wire between P and Q, a student says the wire at the midpoint of P and Q is displaced 2.0 cm from equilibrium as it oscillated, and it moves with an average speed of 0.16 m s^{-1} .

Write an expression for the current supplied to the wire at time t . You may leave a factor of π in your expression.

Ans:

Find the period T of oscillations:

- In one period, the wire moves 2.0 cm 4 times (up, back down, down, back up)
- Distance moved, $d = 8.0 \text{ cm} = 0.080 \text{ m}$
- $T = \frac{d}{v} = \frac{0.080}{0.16} = 0.50 \text{ s}$ [1 mark]

Convert this to angular frequency:

- $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.50} = 4\pi \text{ rad s}^{-1}$ [1 mark]

Write an expression for current:

- $I = I_0 \sin(\omega t)$
- $I = 3.4 \sin(4\pi t)$ [1 mark]

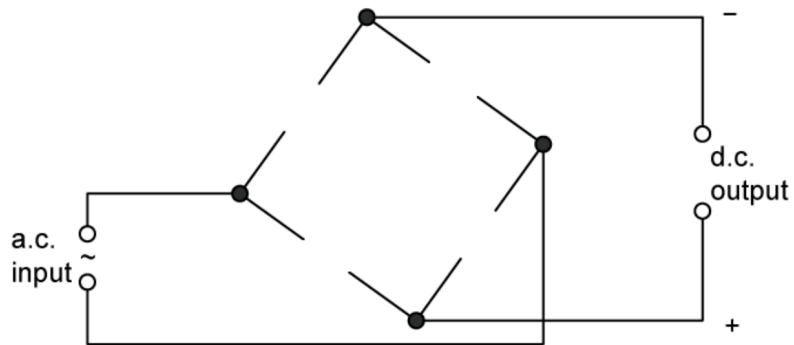


Over one full oscillation, the wire has moved 8.0 cm in total at the average speed of 0.16 m s^{-1} .

In reality, the oscillations of the wire are not symmetrical. Explain why.

- The wire oscillates up and down/ vertically.
- Fleming's left-hand rule shows this.

- Resultant force on the wire when moving upwards < resultant force on the wire when moving downwards
- Because the wire's weight acts downwards.
Resultant force downwards = 40N + weight.



Draw what should be done for smoothing of the pd across the dc output

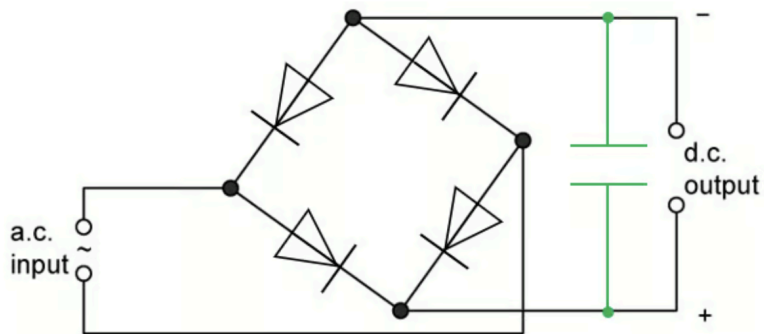


Fig. 1.3 shows the variation with time t of the smoothed potential difference V across the d.c. output.

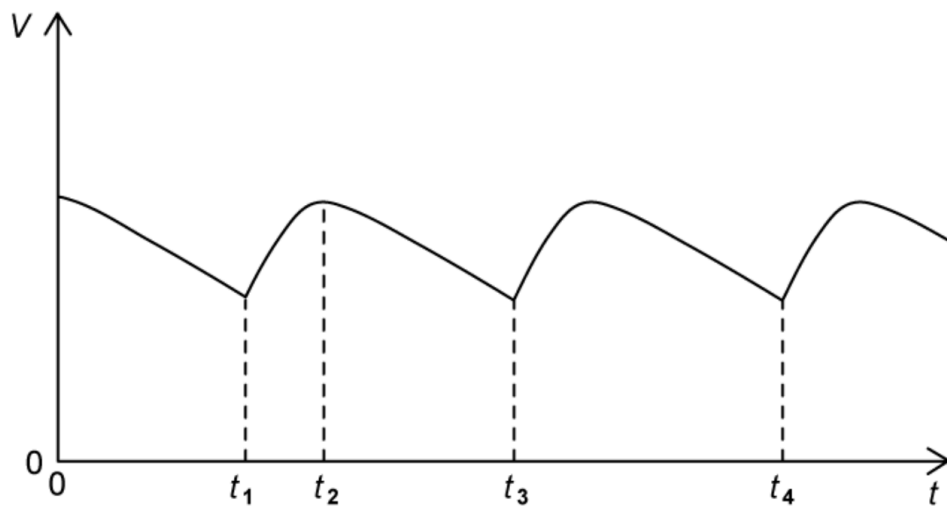


Fig. 1.3

Using Fig. 1.3, state the interval of time during which the capacitor is being charged.

Ans:

$t_1 - t_2$

22. Quantum physics

State observed phenomena that led scientists to believe that EM radiation was a wave.

- Diffraction
- Interference
- Refraction
- Polarisation

State experiments showing that EM radiation can exhibit particle properties.

- Photoelectric effect
- Atomic line spectra

Define photon

- A discrete quantum of electromagnetic radiation.

A heated filament and anode are placed in a vacuum, as shown in Fig 1.1. The anode has a gap through which electrons can pass.

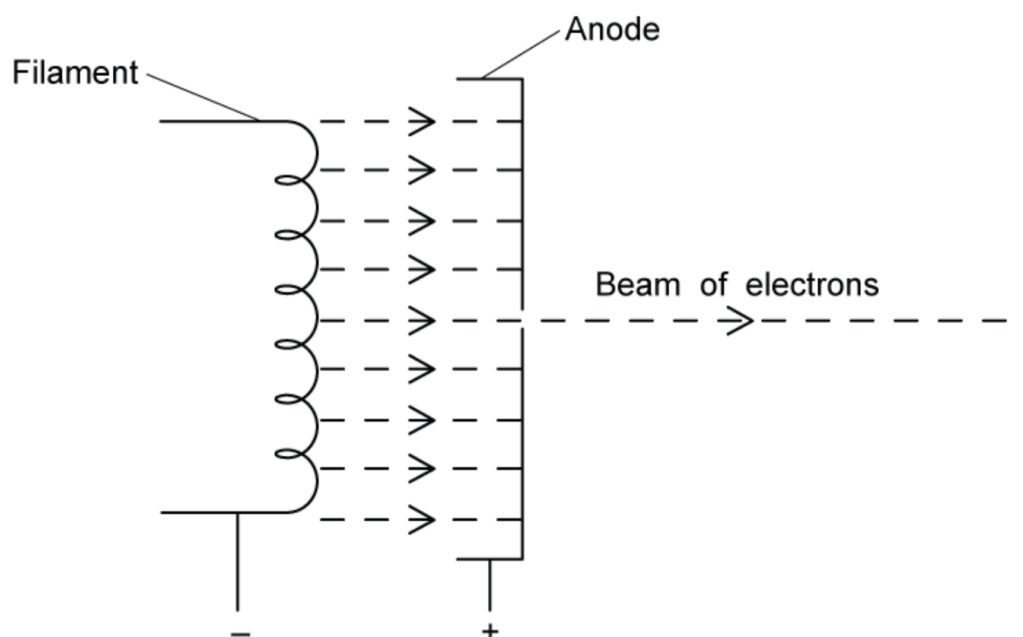


Fig 1.1

A potential difference of 15 V is applied across the filament and anode.

An electron is then emitted from the heated filament with a speed of $1.5 \times 10^6 \text{ m s}^{-1}$.

Calculate its initial kinetic energy in electronvolts.

Ans:

Calculate the kinetic energy, E_k , in joules:

- $E_k = \frac{1}{2} m_e v^2$
- $E_k = \frac{1}{2} \times 9.11 \times 10^{-31} \times (1.5 \times 10^6)^2$ [1 mark]
- $E_k = 1.02 \times 10^{-18} \text{ J}$

Convert this to eV:

- $1 \text{ J} = \left(\frac{1}{1.60 \times 10^{-19}} \right) \text{ eV}$
- $E_k = 1.02 \times 10^{-18} \text{ J} = \frac{1.02 \times 10^{-18}}{1.60 \times 10^{-19}} \text{ eV}$ [1 mark]
- $E_k = 6.4 \text{ eV}$ [1 mark]

Calculate the electron's final kinetic energy in joules, as it exits the gap in the anode.

Ans:

Determine the work done, W by the uniform electric field:

- $W = qV$
 - where q is the charge on the particle and V is the potential across the uniform field
- $W = e \times 15 \text{ V} = 15 \text{ eV}$ [1 mark]

Determine the final kinetic energy, $E_{k,f}$ in eV:

- $E_{k,f} = E_{k,i} + W$
- $E_{k,f} = 6.4 + 15 = 21.4 \text{ eV}$ [1 mark]

Convert this energy to joules:

- $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$
- $E_{k,f} = 21.4 \text{ eV} = 21.4 \times (1.60 \times 10^{-19}) \text{ J}$
- $E_{k,f} = 3.4 \times 10^{-18} \text{ J}$ [1 mark]

Calculate the momentum of this electron after it has passed through the hole in the anode.

Ans:

Rearrange the equation for kinetic energy to make speed the subject:

- $E_k = \frac{1}{2} m_e v^2$
- $v = \sqrt{\frac{2E_k}{m_e}}$

Substitute the known quantities:

- $v = \sqrt{\frac{2 \times (3.4 \times 10^{-18})}{9.11 \times 10^{-31}}}$ [1 mark]
- $v = 2.73 \times 10^6 \text{ m s}^{-1}$

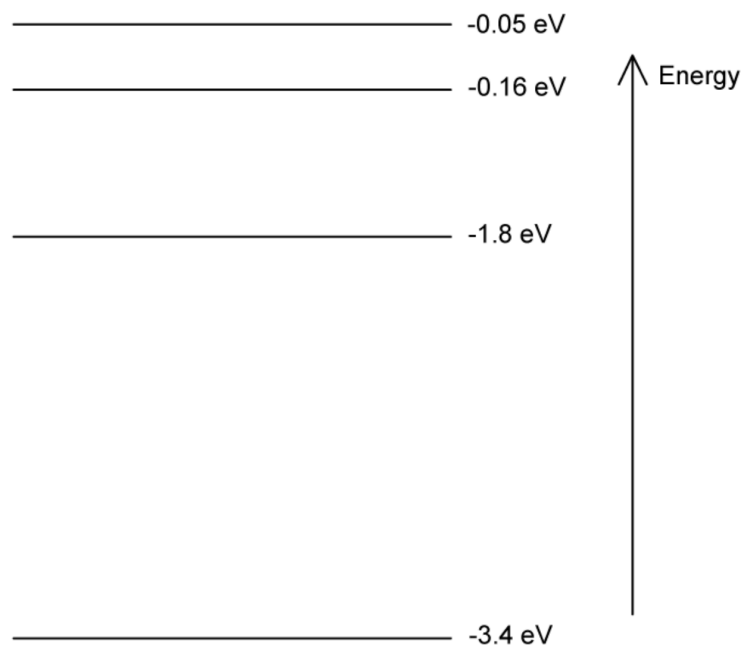
Calculate momentum, p :

- $p = mv = (9.11 \times 10^{-31}) \times (2.73 \times 10^6)$
- $p = 2.49 \times 10^{-24} \text{ kg m s}^{-1} = 2.5 \times 10^{-24} \text{ kg m s}^{-1}$ [1 mark]

NOTE: Can't use $p = E/c$ for this, as that equation only applies for momentum of a photon!!!!

A laser sitting at rest on wheels that are free to move produces 5.00×10^{27} photons. Each photon has a frequency 3.95×10^{14} Hz.

Lasers produce photons of light through a process called stimulated emission. The energy of a photon produced by the laser is equal to the energy difference between the electrons in an excited state and the level below. Fig. 1.1 shows the energy levels of the electrons inside the laser from parts a) and b).



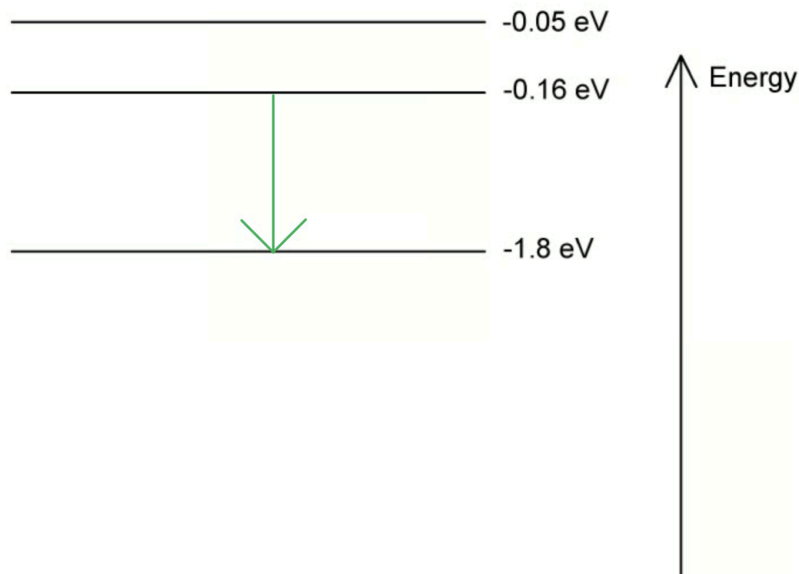
Ans:

Calculate the energy of one photon, E :

- Recall: $E = hf$
- $E = (6.63 \times 10^{-34}) \times (3.95 \times 10^{14})$
- $E = 2.62 \times 10^{-19} \text{ J}$ [1 mark]

Convert the energy in J to energy in eV:

- $E \text{ in eV} = \frac{2.62 \times 10^{-19}}{1.6 \times 10^{-19}}$
- $E \text{ in eV} = 1.64 \text{ eV}$ [1 mark]



$$hf = \phi + \frac{1}{2}mv_{\max}^2$$

Explain the meaning of each term in the photoelectric equation

- hf = energy of a single photon.
- ϕ = minimum energy required to release a photoelectron from the metal surface.
- $\frac{1}{2}mv_{\max}^2$ = maximum kinetic energy of a photoelectron emitted from the metal surface.

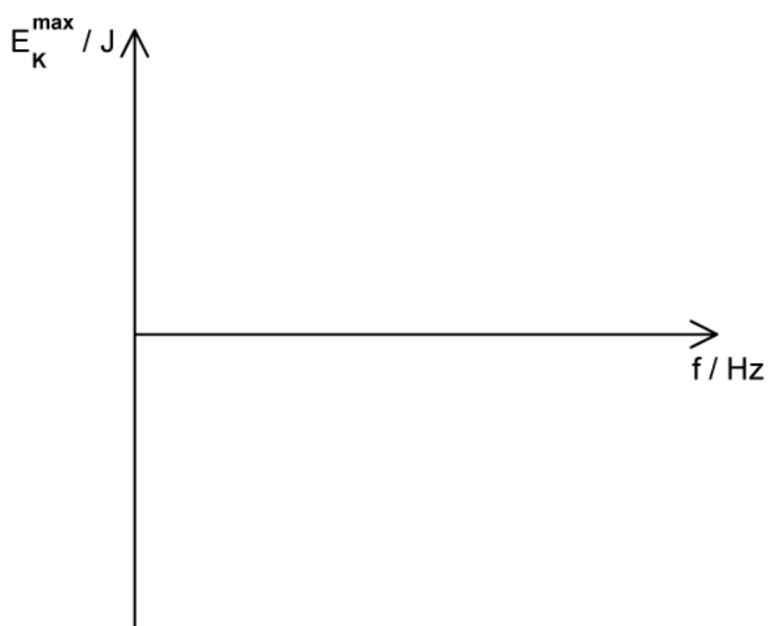
Outline how the photon model of light is used to explain the photoelectric effect.

- Light is made of photons, each carrying an energy $E = hf$.
- Photons are absorbed by electrons in a 1:1 ratio.
- Photoelectrons are released immediately from the metal surface.
- If the electron gains sufficient energy/ energy exceeding the work function (from the photon).

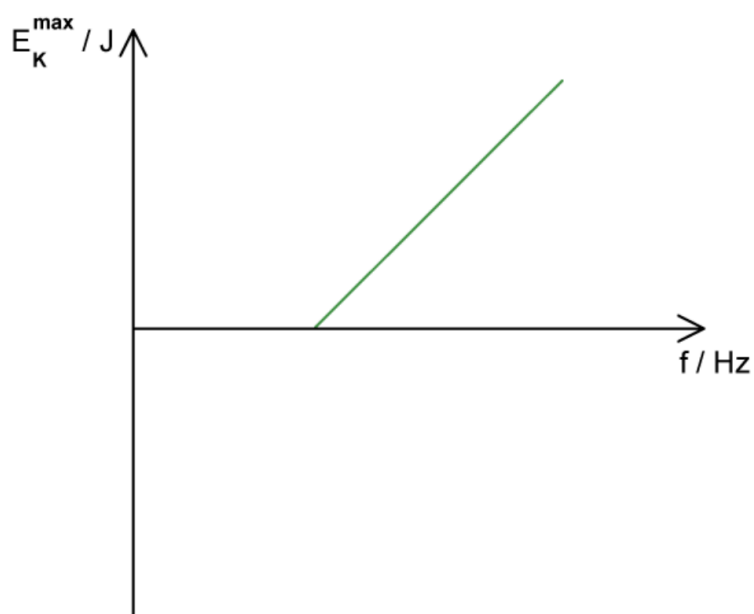
No photoelectron emission is observed from the metal surface when incident light is below a certain frequency. Outline why wave theory for light cannot explain this observation.

- All waves transfer energy regardless of wavelength.
- Over time, enough energy should accumulate/ build up to emit electrons.

Sketch a graph of $E_{K(max)}$ against f on the axes provided.



Ans:



- Straight line drawn with a positive gradient; [1 mark]
- The line intercepts the frequency axis / does not pass through the origin / zero; [1 mark]

Outline what is meant by work function.

- Minimum energy required to emit an electron from the metal surface.

- With zero/ no kinetic energy.

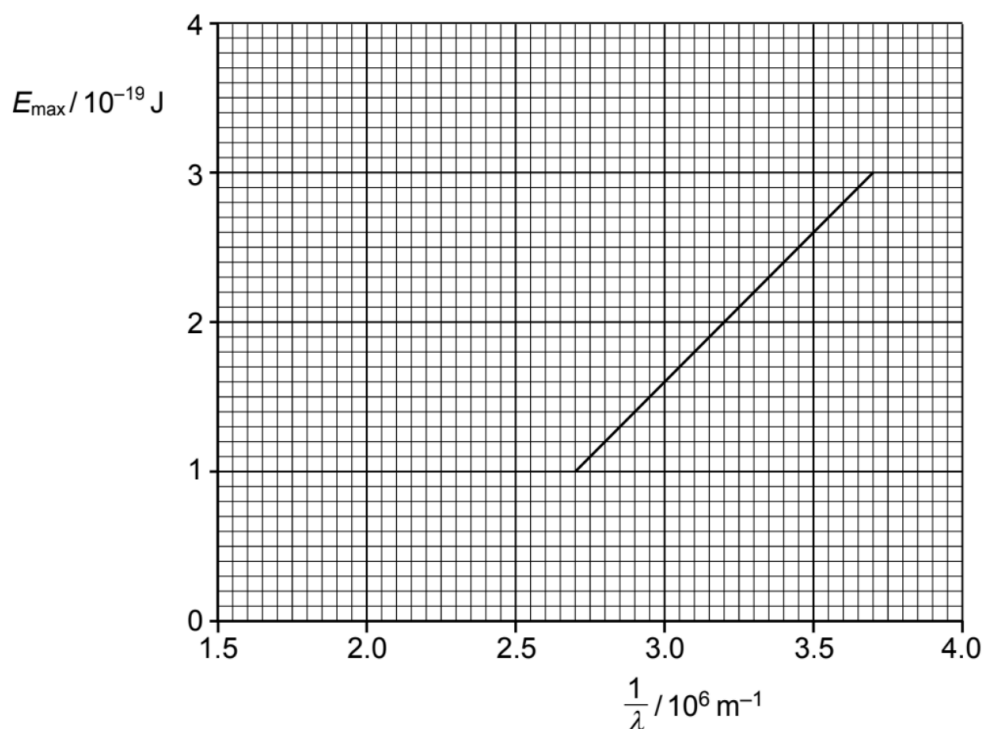
Outline what is meant by threshold frequency.

- Minimum frequency of an incident photon.
- Required to emit an electron from the metal surface.

Describe phenomena associated with the photoelectric effect that cannot be explained using a wave theory of light.

- No photoelectrons are emitted from metal surface below the threshold frequency.
- The emission of a photoelectrons from the metal plate is instantaneous.
- Maximum kinetic energy of emitted photoelectrons is dependent on frequency of incident photon/ light.
- Maximum kinetic energy of emitted photoelectrons is independent of intensity of incident radiation/ light.

The variation with $\frac{1}{\lambda}$ of $E_{\max}$ for the metal surface is shown in Fig. 8.1.



Use Fig. 8.1 to determine the magnitude of λ_0 .

$\lambda_0 = \dots\dots\dots \text{m [1]}$

Ans:

Draw lines on the graph to extend the given line to the x-axis:

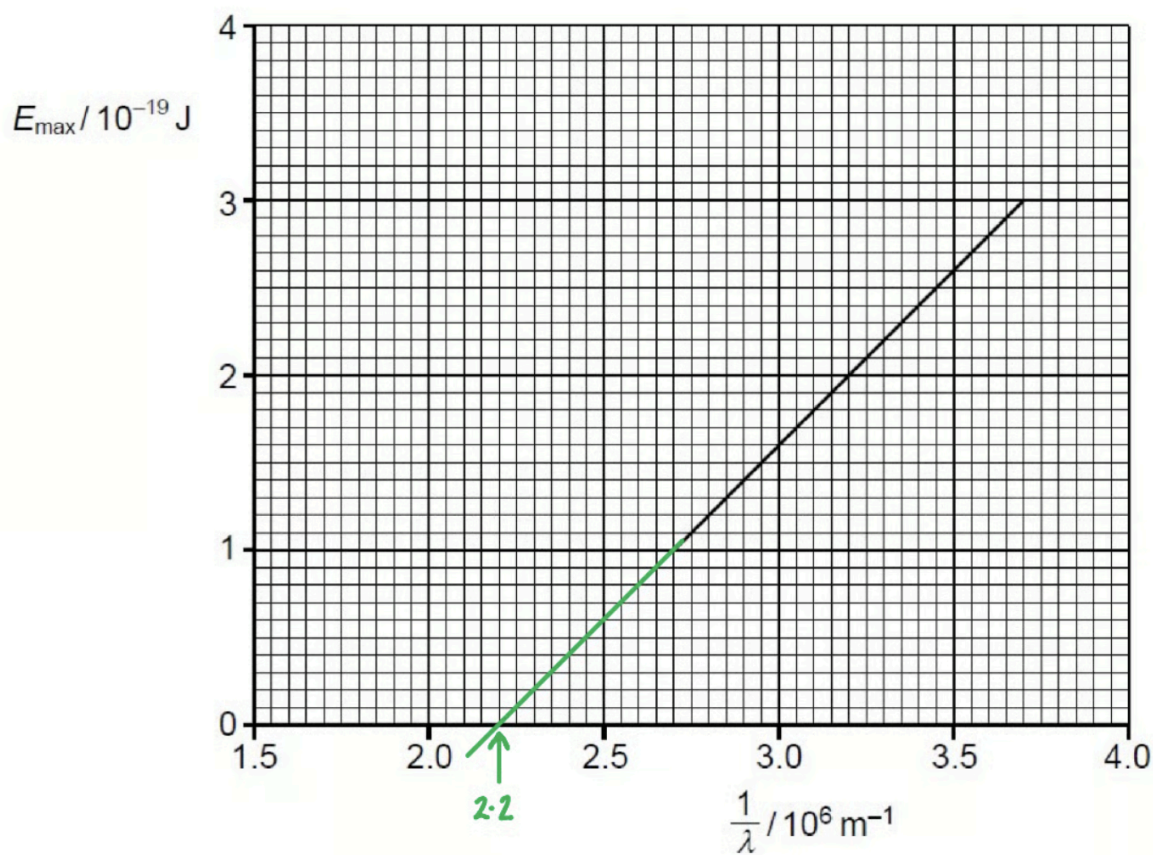


Fig. 8.1

- The line intercepts the x-axis at $2.2 \times 10^6 \text{ m}^{-1}$
- So, $\frac{1}{\lambda_0} = 2.2 \times 10^6 \text{ m}^{-1}$
- Hence, $\lambda_0 = 4.5 \times 10^{-7} \text{ m}^{-1}$ [1 mark]

Use the gradient of Fig. 8.1 to determine a value for the Planck constant h . Show your working.

$h = \dots\dots\dots \text{ J s}$ [3]

Ans:

Calculate the gradient of the line, m :

- $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(1 \times 10^{-19}) - (3 \times 10^{-19})}{(2.7 \times 10^6) - (3.7 \times 10^6)}$
- $m = 2 \times 10^{-25}$ [1 mark]

Rearrange to make Planck's constant h the subject and substitute for c into the expression for the gradient, m :

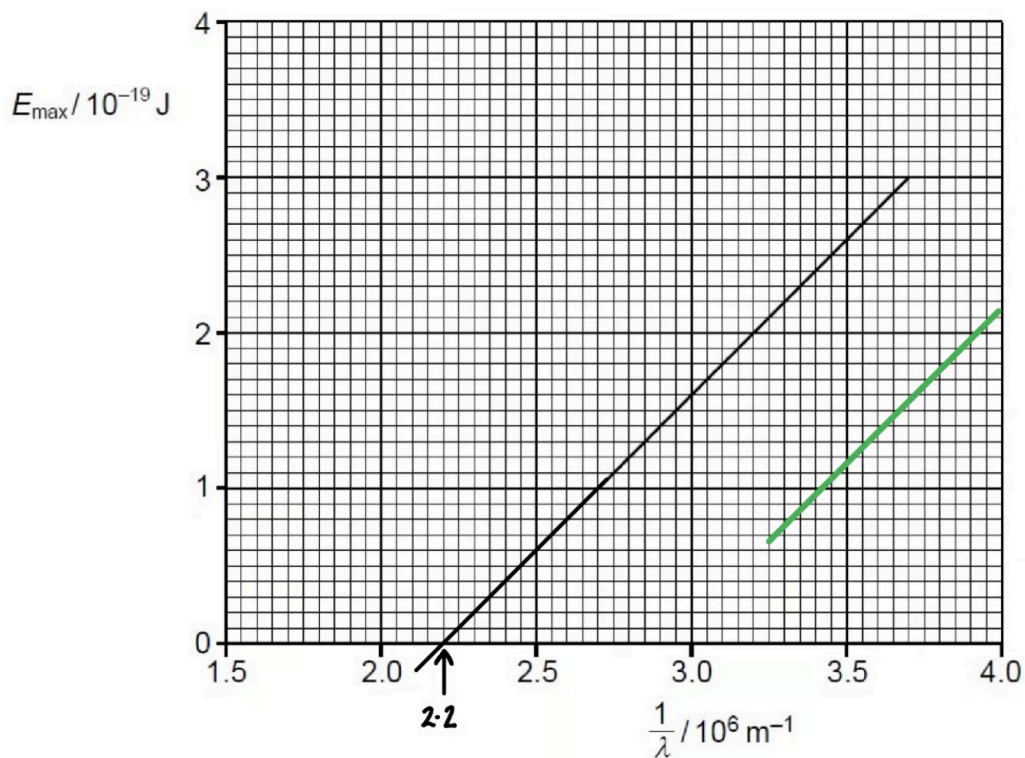
- $m = hc$
- $h = \frac{m}{c}$
- $h = \frac{(2 \times 10^{-25})}{(3 \times 10^8)}$
- $h = 6.66 \times 10^{-34} \text{ J s} = 6.7 \times 10^{-34} \text{ J s (2 s.f.)}$ [1 mark]

The metal surface in **(b)** becomes oxidised.

Photoelectric emission is still observed but the work function energy is increased.

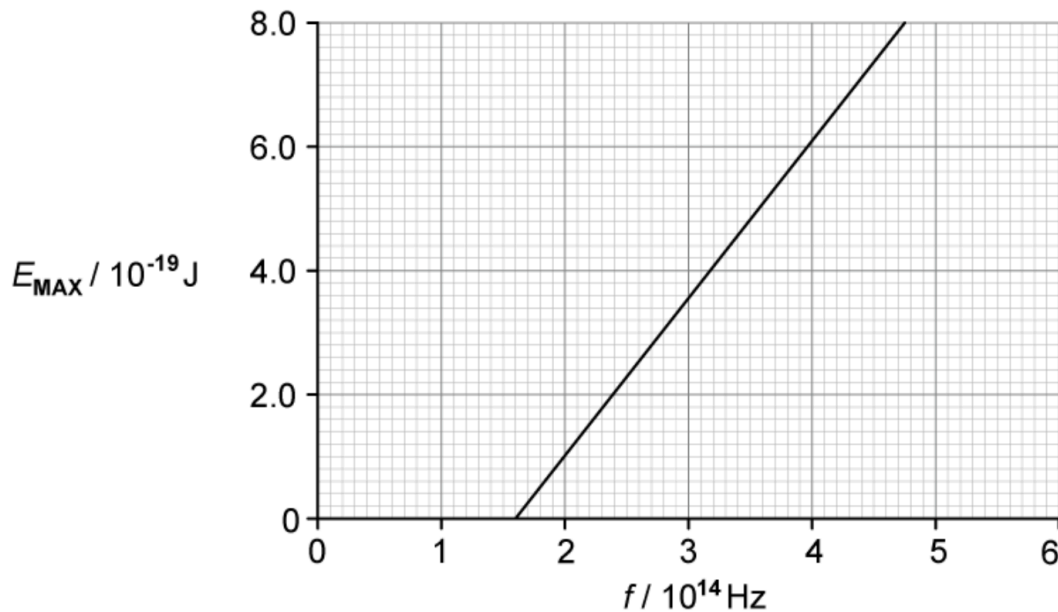
On Fig. 8.1, draw a line to show the variation with $\frac{1}{\lambda}$ of E_{max} for the oxidized surface.

Ans:



- The line has the same gradient as the given line; [1 mark]
- The line is straight, drawn with a ruler, with a positive gradient and when extrapolated intercepts the x -axis at a value greater than $2.2 \times 10^6 \text{ m}^{-1}$ [1 mark]

The variation with f of the maximum kinetic energy E_{MAX} of the emitted electrons is shown in Fig. 1.1.



Describe conclusions that can be drawn from the graph (quantitative or qualitative)

- There is a threshold frequency below which no photoelectrons are emitted.
OR the threshold frequency is $1.6 \times 10^{14} \text{ Hz}$.
- The gradient of the line is the Planck's constant (h).
OR the gradient of the line is $6.63 \times 10^{-34} \text{ J s}$.
- The work function of the metal is calculated to be $1.06 \times 10^{-19} \text{ J}$.
- E_{max} increases linearly with increasing frequency.

State and explain how the graph would compare when:

a. The same metal is used, but with EM radiation of intensity $I/2$.

- Look the same/ line would be unchanged.
- Because the photons have the same energy/ photon energy is independent of intensity.

b. A different metal is used, but the intensity I of the radiation is the same.

- Have the same gradient, but different x -intercept.
- Because the threshold frequency would be different.

The frequency of radiation incident on a surface is decreased, while the power remains constant. State and explain the effect of this change on:

a. The maximum kinetic energy of the photoelectron.

- The photon energy decreases, for the same work function.
- So the maximum kinetic energy will decrease.

b. The rate of emission of the photoelectron.

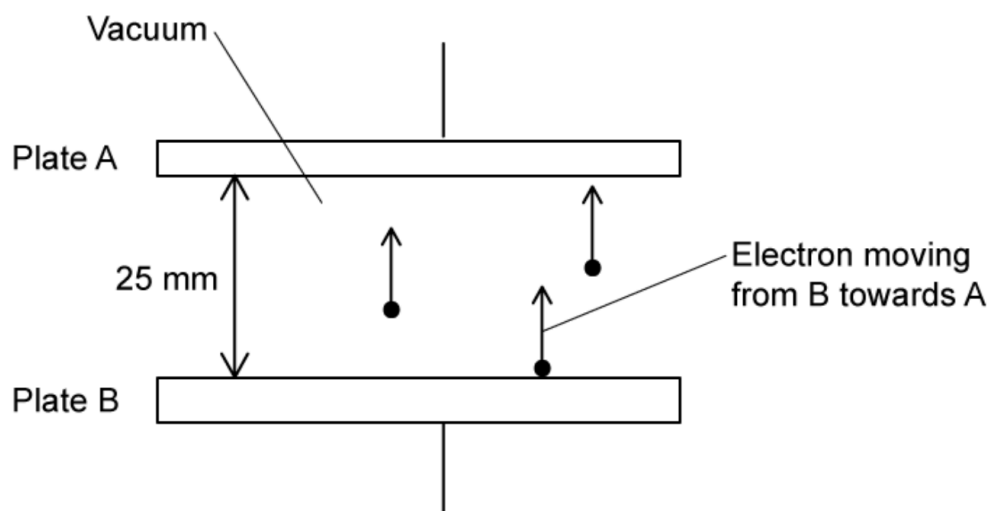
- The lower the photon energy, with the same power, will increase the number of photons per unit time.

$$\text{Number per unit time} = \frac{\text{Power}}{\text{Energy}} = \frac{\text{Power}}{KE_{\text{max}}}$$

The smaller KE_{max} then the higher the number per unit time.

A certain metal with a work function of 2.2 eV is selected to create two parallel plates. These are connected in a circuit to a sensitive ammeter and a power source, such that a uniform electric field of 140 N C^{-1} exists between them. Fig. 1.2 shows the two plates, labelled A and B, separated by 25 mm in a vacuum.

Electromagnetic radiation of wavelength 255 nm is incident on plate B, such that photoelectrons are emitted from B and move towards A. The ammeter detects a photocurrent in the circuit.



State the direction of the electric field between the plates, given the photocurrent is at a maximum. Explain your answer.

- Maximum photocurrent means all emitted electrons are collected at plate A.
- If all emitted photoelectrons are collected, plate A must be at a higher potential than plate B; plate A must be positively charged.

- Hence, the direction of the electric field is from plate A to plate B.

NOTE: the only way you can ensure maximum photocurrent, is to attract the electrons to a positively charged region (A). if the field were pointing the other way, a photocurrent may still be detected, but not at a maximum. Some electrons may be emitted with too little KE to overcome the opposing electric field.

Show that the maximum possible speed of the photoelectron as they reach plate A is approximately $1.5 \times 10^6 \text{ m s}^{-1}$.

Calculate the energy available from incident light:

- $hf = h \times \frac{c}{\lambda} = (6.63 \times 10^{-34}) \times \left(\frac{3 \times 10^8}{255 \times 10^{-9}} \right) = 7.8 \times 10^{-19} \text{ J}$ [1 mark]

Calculate the maximum kinetic energy of photoelectrons released to the surface:

- $hf = \phi + KE_{\text{max}}$ so $KE_{\text{max}} = hf - \phi$
- $KE_{\text{max}} = (7.8 \times 10^{-19}) - (3.5 \times 10^{-19}) = 4.3 \times 10^{-19} \text{ J}$ [1 mark]

Calculate the potential difference between the plates, V:

- The uniform electric field strength between parallel plates $E = \frac{V}{d}$
- The potential difference $V = E \times d = 140 \times (25 \times 10^{-3}) = 3.5 \text{ V}$ [1 mark]

Calculate the kinetic energy transferred due to the potential difference between the plates:

- The potential difference between the plates, $V = \frac{W}{e}$
- The work done on the photoelectrons contributes to their KE, so $W = KE = e \times V$
- $KE = (1.6 \times 10^{-19}) \times 3.5 = 5.6 \times 10^{-19} \text{ J}$ [1 mark]

Calculate the total maximum kinetic energy of photoelectrons reaching plate A:

- Total maximum $KE_{\text{total}} = KE_{\text{max}}$ from incident photons + KE transferred from the potential difference
- Total maximum $KE_{\text{total}} = (4.3 \times 10^{-19}) + (5.6 \times 10^{-19}) = 9.9 \times 10^{-19} \text{ J}$ [1 mark]

Calculate the maximum speed of photoelectrons:

- $KE_{total} = \frac{1}{2} m(v_{max})^2$
- $9.9 \times 10^{-19} = \frac{1}{2} \times (9.11 \times 10^{-31}) \times (v_{max})^2$
- $19.8 \times 10^{-19} = (9.11 \times 10^{-31}) \times (v_{max})^2$
- $(v_{max})^2 = 2.17 \times 10^{12}$
- $v_{max} = \sqrt{2.17 \times 10^{12}} = 1.474 \times 10^6 \text{ m s}^{-1} = 1.5 \times 10^6 \text{ m s}^{-1} (2 \text{ s. f.})$ [1 mark]

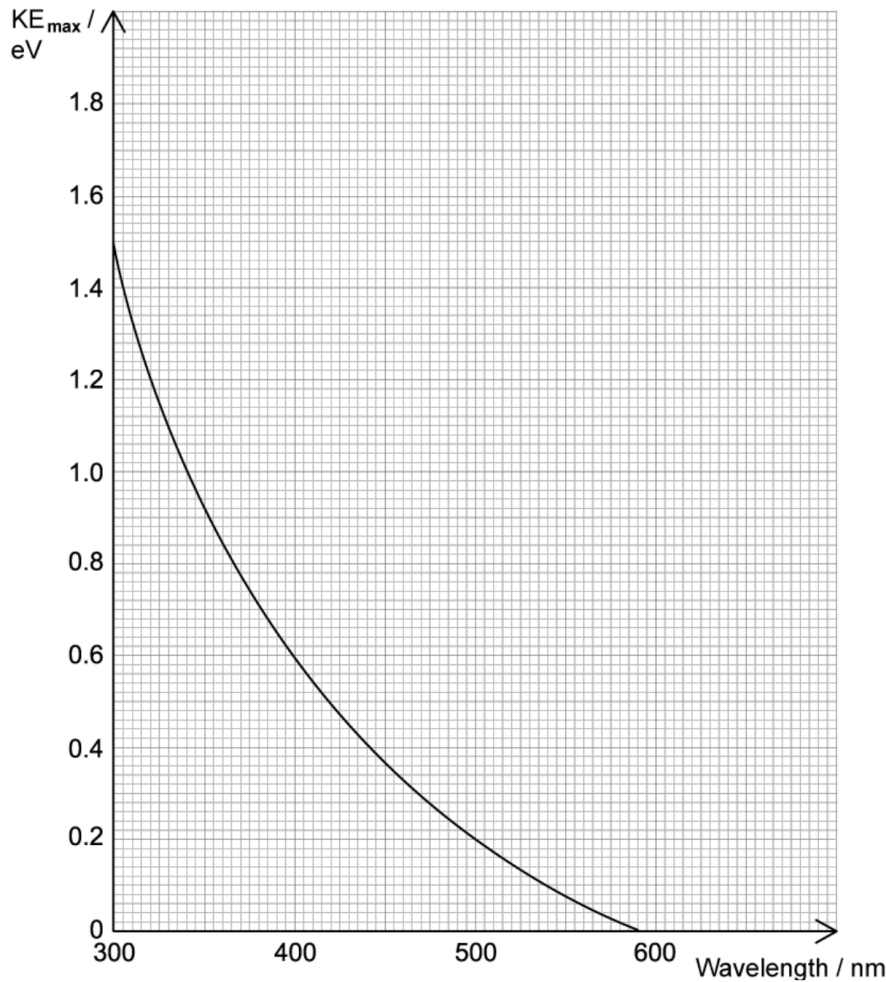
NOTE: Maximum speed of photoelectrons = speed due to maximum kinetic energy on release from metal surface + speed due to energy supplied by traveling across a potential difference.

Suggest and explain an experimental change to the circuit, that would reduce the photocurrent measured.

- The terminals of the power supply should be reversed/ make plate A negatively charged and B positively charged/ reverse direction of electric field.
- This reversal would make photoelectrons do work against the electric field between the plates/ electrons would need to do work to cross the potential difference between the plates.
- This would reduce the photocurrent, as only photoelectrons with sufficient energy would be able to reach plate A/ could cross the potential difference.

If the direction of the electric field is reversed (by reversing the terminals of the power supply) then there will be a force opposing the motion of every photoelectron emitted from the surface of plate B. Therefore, electrons would need to do work **against** this force; only the most energetic photoelectrons (those nearest the surface of the metal!) would be able to cross the space between the plates. The overall effect would thus be a reduction in the photocurrent.

The maximum kinetic energy, KE_{max} of photoelectrons varies with the wavelength of incident light on a metal surface. This is shown in Fig. 1.1 below:



Ans:

Use Fig. 1.1 to find the wavelength when $KE_{max} = 0$:

- Photoelectrons are only released to the surface if light is incident at the threshold frequency (or threshold wavelength, λ_0)
- $\lambda_0 = 590$ nm when the $KE_{max} = 0$ from Fig. 1.1; [1 mark]

- $hf = \phi + KE_{max}$

- When $KE_{max} = 0$, the work function $\phi = hf_0$

- $\phi = hf_0 = h \times \frac{c}{\lambda_0} = (6.63 \times 10^{-34}) \times \left(\frac{3 \times 10^8}{590 \times 10^{-9}} \right) = 3.4 \times 10^{-19} \text{ J}$ [1 mark]

Calculate the work function in electronvolts, ϕ :

- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
- $\phi = (3.4 \times 10^{-19}) \div (1.6 \times 10^{-19}) = 2.1 \text{ eV}$ [1 mark]

NOTE: $\phi = hf_0 = hc/\lambda_0$

A student uses the shape of the curve to state that maximum KE is inversely proportional to the incident wavelength. By referring to the photoelectric equation, discuss the validity of this statement.

- Photoelectric equation: $hf = \phi + E_{kmax}$ OR $E_{kmax} = hf - \phi$
- In terms of λ : $E_{kmax} = hc/\lambda - \phi$
- The student's statement is not valid.
- Inverse proportionality means $E_{kmax} \times \lambda = \text{constant}$, which is not the case here OR the work function ϕ is a term which cannot be ignored in the relationship.

NOTE: an inversely proportional graph does not cross any axes, so this cannot be inverse proportionality.

Calculate the de Broglie wavelength of photoelectrons emitted from the metal surface for incident light of wavelength of 500nm.

Ans:

- If the wavelength of incident light is 500 nm then the KE_{max} of photoelectrons is 0.2 eV; [1 mark]

Convert the from eV to J:

- $KE_{max} = 0.2 \text{ eV} = 0.2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-20} \text{ J}$ [1 mark]

Calculate the velocity of the emitted photoelectrons:

- $KE_{max} = \frac{1}{2} m (v_{max})^2$
- $3.2 \times 10^{-20} = \frac{1}{2} \times (9.11 \times 10^{-31}) \times (v_{max})^2$
- $6.4 \times 10^{-20} = (9.11 \times 10^{-31}) \times (v_{max})^2$
- $(v_{max})^2 = 7.03 \times 10^{10}$
- $v_{max} = \sqrt{7.03 \times 10^{10}} = 2.7 \times 10^5 \text{ m s}^{-1}$ [1 mark]

Calculate the de Broglie wavelength of the emitted photoelectrons:

- $\lambda = \frac{h}{mv}$
- $\lambda = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31}) \times (2.7 \times 10^5)} = 2.7 \times 10^{-9} \text{ m}$ [1 mark]

[Total: 4 marks]

Discuss the experimental changes that would reduce the minimum de Broglie wavelength of the emitted photoelectrons.

- The maximum velocity/ momentum of photoelectrons must be increased.
- Increase in maximum velocity is achieved by an increase in maximum $E_{k\max}$.
- $E_{k\max}$ of emitted photoelectrons can be increased by using a metal with smaller work function energy for the same incident light source.
- $E_{k\max}$ of emitted photoelectrons can also be increased by increasing the frequency/ decreasing the wavelength of the light source, for the same metal surface.

Outline the meaning of the de Broglie wavelength

- All particles have a wavelength associated with them.
- Wavelength is inversely proportional to the momentum of the particle:
 $\lambda = h/p$.

NOTE:

- electron diffraction provides evidence that a moving electron behaves as a wave.
- If speed of electrons increases, radius of concentric rings observed on fluorescent screen decreases.

A simplified diagram of an electron gun and bulb is shown in Fig 1.1 below:

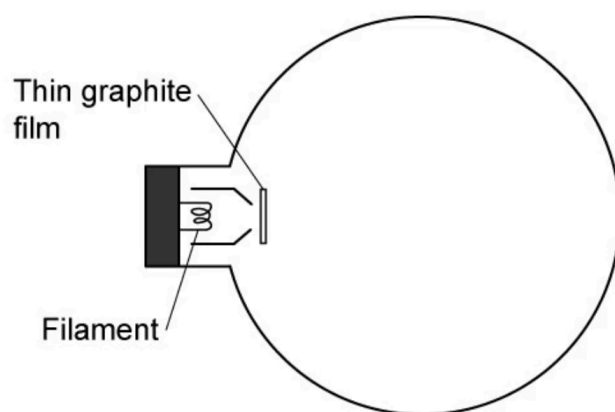


Fig 1.1

The electrons are produced from a very hot filament, and are directed toward a thin graphite film, all of which are set up inside an evacuated glass bulb.

A very characteristic pattern is formed at the back of the bulb, which demonstrates electrons have been diffracted.

Describe the pattern observed at the back of the bulb

- Bright and dark concentric rings

State why the bulb must be evacuated

- To ensure electrons don't collide with air particles/ so diffraction pattern is not distorted due to

Explain why the electrons diffract

- Electrons have a de Broglie wavelength
- Which matches the gap width between atoms of the crystalline material OR Matches the diameter of the atom

A student wonders if high energy electrons could be used to investigate the nature of protons which have an estimated diameter of $0.84 \times 10^{-15} \text{ m}$.

Calculate the momentum of the electrons that would be suitable for this investigation.

Ans:

- Maximum diffraction of a wave around a barrier occurs when the wavelength is similar to the size of the barrier
- Therefore, the de Broglie wavelength of the electrons needs to equal the diameter $d = 0.84 \times 10^{-15} \text{ m}$ [1 mark]

Substitute known values into the de Broglie equation:

- $\lambda = \frac{h}{p}$
- $0.84 \times 10^{-15} = \frac{6.63 \times 10^{-34}}{p}$ [1 mark]

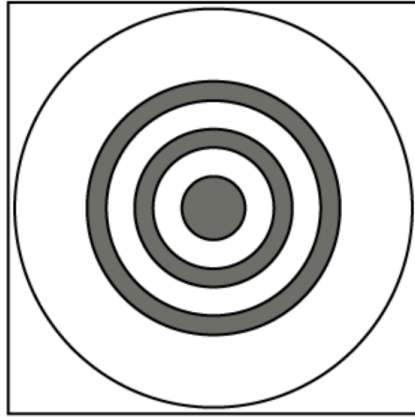
Rearrange and solve for momentum, p :

- $p = \frac{6.63 \times 10^{-34}}{0.84 \times 10^{-15}} = 7.9 \times 10^{-19} \text{ kg m s}^{-1}$ [1 mark]

NOTE: the speed of anything should be less than or equal to than speed of light for that experiment to be possible.

Electrons accelerate through a potential difference, pass through a thin crystal and are then incident on a fluorescent screen.

The pattern in Fig. 1.1 is observed on the fluorescent screen.



The potential difference used to accelerate the electron is changed.

The new pattern observed on the screen is shown in Fig. 1.2.

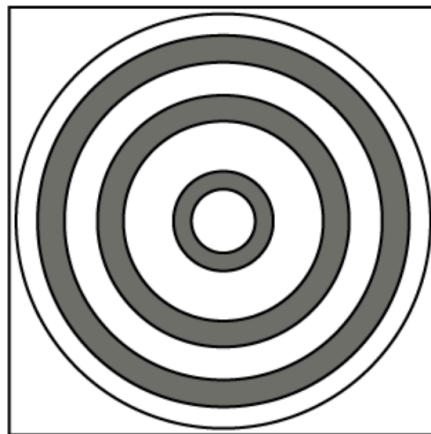
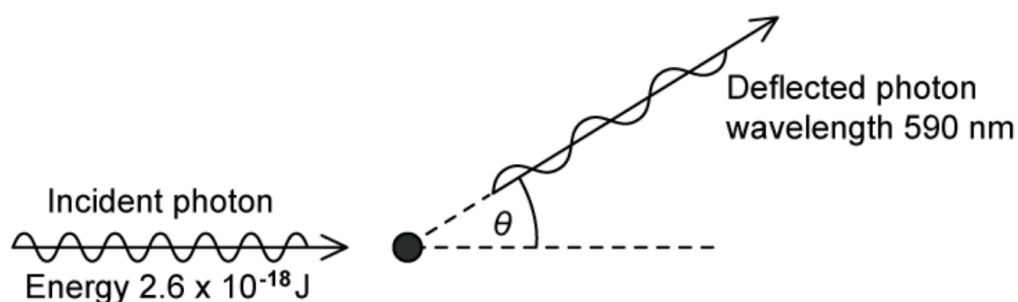


Fig 1.2

State and explain the change that has been made to the potential difference to create the pattern shown in Fig. 1.2.

- Wavelength has increased OR the electron had a lower momentum.
- So the accelerating p.d. was decreased.

A gamma-ray photon of energy $2.6 \times 10^{-18} \text{ J}$ is incident on an isolated stationary electron, as illustrated in Fig. 1.1.



Calculate the energy of the deflected photon

Ans:

Use the equation for calculating photon energy, E :

- $E = \frac{hc}{\lambda}$
- $E = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^8)}{(590 \times 10^{-9})}$ [1 mark]
- $E = 3.37 \times 10^{-19} \text{ J}$ [1 mark]

Explain why the magnitude of the final momentum of the electron is not equal to the change in magnitude of the momentum of the photon.

- Momentum is a vector quantity.
- So the momentum must be considered in 2 directions OR
The direction of the electron changes so magnitude alone cannot be considered.

An experiment to investigate how the de Broglie wavelength λ of an electron varies with its velocity v is carried out. The results from this experiment are shown in Table 1.1.

$v / 10^4 \text{ km s}^{-1}$	$\lambda / 10^{-11} \text{ m}$
1.4	4.9
2.5	2.8
3.5	2.1

Using the data in table, and accepted value of Planck's constant, comment on accuracy of results.

Method 1: Calculate the mean constant of proportionality from part (a)

- From part (a), $\lambda v = \text{constant}$
- The mean constant from the data is $\frac{(6.9 + 7.0 + 7.4) \times 10^{-4}}{3} = 7.1 \times 10^{-4}$ [1 mark]

Use the mean constant λv to calculate a value for Planck's constant:

- Using de Broglie's equation, $h = m \times \lambda v$
- $h = (9.11 \times 10^{-31}) \times (7.1 \times 10^{-4}) = 6.47 \times 10^{-34} \text{ J s (3 s.f.)}$ [1 mark]

Calculate the percentage difference to the accepted value for Planck's constant:

- The accepted value for Planck's constant is $6.63 \times 10^{-34} \text{ J s}$
- The percentage difference $\Delta\% = \frac{(6.63 - 6.47) \times 10^{-34}}{6.63 \times 10^{-34}} \times 100 = 2\%$ [1 mark]

Comment on the accuracy of the experiment:

- The percentage difference between the value of h suggested by the experiment and the accepted value is small **AND** so the experimental results are accurate; [1 mark]

Method 2: Calculate Planck's constant for each pair of values of λ and v

- Using de Broglie's equation, $h = m \times \lambda v$
- The constant from the pair of values in the first row of Table 1.1, $\lambda v = 6.9 \times 10^{-4}$
 - $h = (9.11 \times 10^{-31}) \times (6.9 \times 10^{-4}) = 6.29 \times 10^{-34} \text{ (J s)}$
- The constant from the pair of values in the second row of Table 1.1, $\lambda v = 7.0 \times 10^{-4}$
 - $h = (9.11 \times 10^{-31}) \times (7.0 \times 10^{-4}) = 6.38 \times 10^{-34} \text{ (J s)}$
- The constant from the pair of values in the third row of Table 1.1, $\lambda v = 7.4 \times 10^{-4}$ so
 - $h = (9.11 \times 10^{-31}) \times (7.4 \times 10^{-4}) = 6.74 \times 10^{-34} \text{ (J s)}$
- All values correctly calculated; [1 mark]

Calculate the mean value of Planck's constant from the data:

- The mean value of Planck's constant is $\frac{(6.29 + 6.38 + 6.74) \times 10^{-34}}{3} = 6.47 \times 10^{-34} \text{ (J s)}$ [1 mark]

Calculate the percentage difference to the accepted value for Planck's constant:

- The accepted value for Planck's constant is $6.63 \times 10^{-34} \text{ J s}$
- The percentage difference $\Delta\% = \frac{(6.63 - 6.47) \times 10^{-34}}{6.63 \times 10^{-34}} \times 100 = 2\%$ [1 mark]

Comment on the accuracy of the experiment:

- The percentage difference between the value of h suggested by the experiment and the accepted value is small **AND** so the experimental results are accurate; [1 mark]

The experiment required electrons to travel at very high speeds. To achieve this, they were first accelerated through a very high voltage, V .

Show that the de Broglie wavelength λ of these electrons is related to the accelerating voltage V by the expression:

$$\lambda \propto \frac{1}{\sqrt{V}}$$

Consider energy transfers for the electron:

- The energy transferred (or work done) W on a particle with charge q as it moves across a potential difference V is given by $W = qV$
- For an electron, $q = e$ so the energy transferred by a potential difference is given by eV
- The energy is transferred is to the electron's kinetic energy, $E_k = \frac{1}{2}mv^2$
- Therefore, $eV = \frac{1}{2}mv^2$ [1 mark]

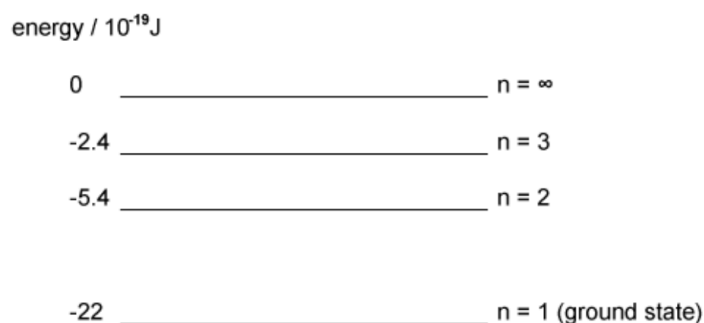
Substitute the de Broglie wavelength:

- de Broglie wavelength $\lambda = \frac{h}{mv}$, therefore $v = \frac{h}{\lambda m}$
- Therefore, $eV = \frac{1}{2}m\left(\frac{h}{\lambda m}\right)^2 = \frac{1}{2}m\left(\frac{h^2}{\lambda^2 m^2}\right) = \frac{h^2}{2m\lambda^2}$ [1 mark]

Deduce the proportionality between λ and V :

- $eV = \frac{h^2}{2m\lambda^2}$
- $\lambda^2 = \frac{h^2}{2meV}$
- $\lambda = h\sqrt{\frac{1}{2meV}}$ therefore $\lambda \propto \frac{1}{\sqrt{V}}$ [1 mark]

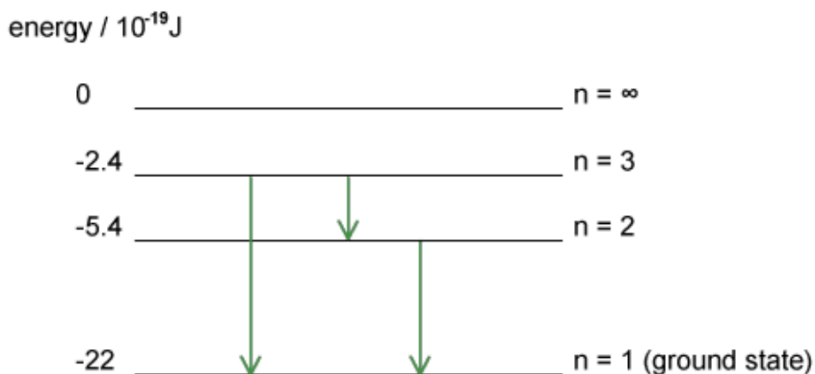
Fig 1.1 shows some of the energy levels of the hydrogen atom.



When electrons in a sample of hydrogen atoms relax from $n = 3$, multiple frequencies of photons are emitted.

Sketch arrows on Fig 1.1 to show the transitions responsible for these photons.

Ans:



Calculate the wavelength of the photon with the smallest energy.

Change in energy:

- Smallest energy transition is from $n = 3$ to $n = 2$
- $\Delta E = E_3 - E_2 = (-2.4 \times 10^{-19}) - (-5.4 \times 10^{-19}) = 3.0 \times 10^{-19} \text{ J}$ [1 mark]

Wavelength calculation:

- $\Delta E = hf = \frac{hc}{\lambda}$ [1 mark]
- $\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^8)}{3.0 \times 10^{-19}}$ [1 mark]
- $\lambda = 6.6 \times 10^{-7} \text{ m}$ [1 mark]

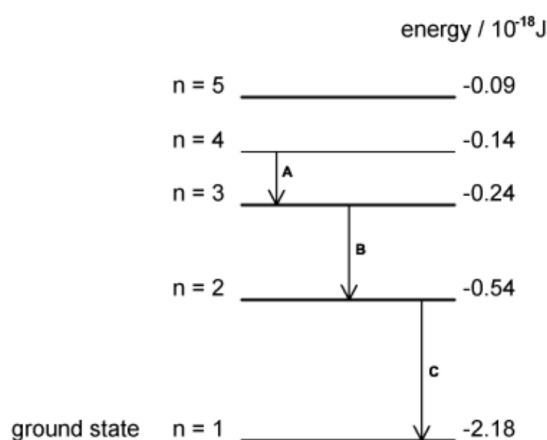
A hydrogen atom can be excited by absorption of a photon. Explain why, for a particular transition, the photon must have an exact amount of energy.

- Energy is needed for electron to move to higher energy level.
- For a transition/ excitation/ change of energy levels, an exact amount of energy is required.
- Photon's energy must be exactly equal to the difference between the energy levels.
- All of the photon's energy is absorbed in a one-to-one interaction.

State the meaning of the term 'ionisation'.

- When an electron gains enough energy to be completely removed from the atom.

An excited hydrogen atom can emit photons of certain discrete frequencies in various regions of the electromagnetic spectrum. Three possible transitions are shown in Fig 1.2.



State the minimum amount of energy required to ionise an electron from the ground level of hydrogen.

- 2.18×10^{-18} J

Remember to use the diagram given earlier in the question to help you answer this. The ionisation level of an atom is defined as having an energy value of 0 J, so the energy difference is 0 J minus the energy of the ground state, which is -2.18×10^{-18} J.

Explain why an emission spectrum is not continuous.

- Only certain wavelengths are emitted by the element.
- Corresponding to certain energy level transitions of the element.

Explain why an absorption spectrum is not continuous.

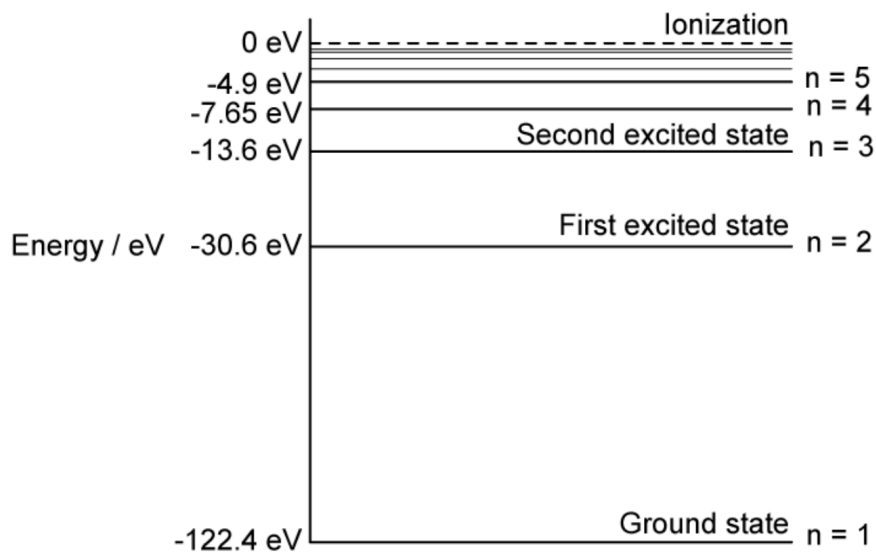
- Only certain wavelengths are missing from the spectrum.
- These are absorbed by the unknown gas.

Explain how absorption spectrum can be used to identify an unknown gas.

- Each element has a characteristic absorption spectrum/ absorbs characteristic wavelengths.
- Compare the unknown absorption spectrum to the absorption spectra for different elements.

NOTE: absorption spectrum = type of spectrum formed when white light is passed through a low density sample of unknown gas.

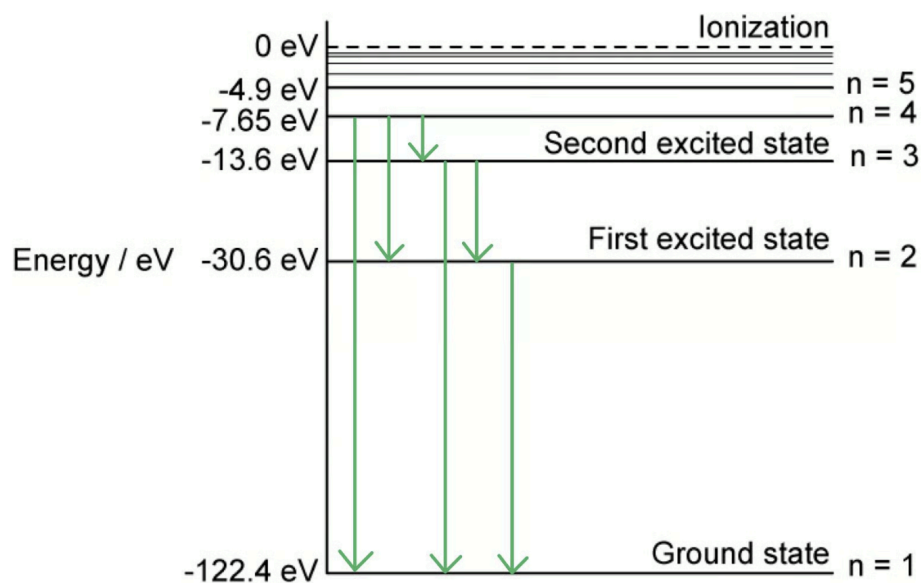
The energy levels of a hydrogen atom are shown in Fig. 1.1.



When an electron of energy $1.84 \times 10^{-17} \text{ J}$ collides with a hydrogen atom, photons of six different energies are emitted.

Sketch arrows on Fig. 1.1 to show the transitions responsible for these photons.

Ans:



Calculate the wavelength of the photon with the smallest energy.

Calculate the smallest change in energy in J from $n = 4$ to $n = 3$:

- $\Delta E = E_4 - E_3$
- $\Delta E = (-7.65) - (-13.6) = 5.95 \text{ eV}$
- $\Delta E = 5.95 \times (1.602 \times 10^{-19}) = 9.53 \times 10^{-19} \text{ J}$ [1 mark]

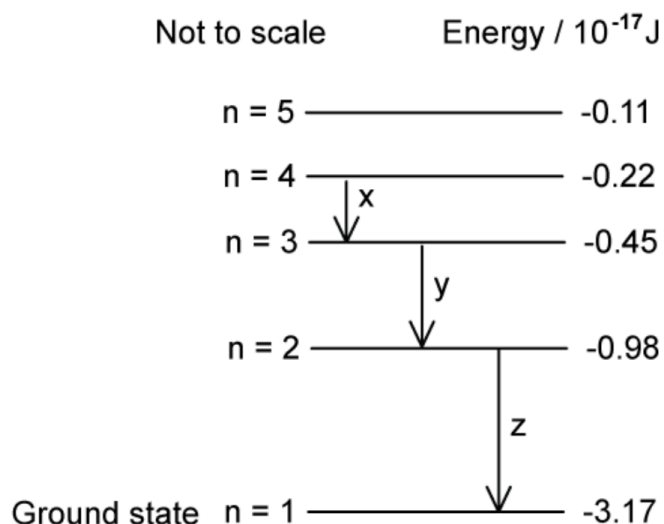
Use the photoelectric equation to calculate the wavelength of the photon:

- $\Delta E = hf = \frac{hc}{\lambda}$
- $\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^8)}{(9.53 \times 10^{-19})}$ [1 mark]
- $\lambda = 2.087 \times 10^{-7} \text{ m} = 2.09 \times 10^{-7} \text{ m (3 s.f.)}$ [1 mark]

Explain why emitted photons have discrete frequencies

- EM radiation is emitted when an electron moves from higher to lower energy level.
- Emitted photon is given a discrete/ specific amount of energy.
- Photon energy is directly proportional to frequency: $\Delta E = hf$, so emitted photons have discrete frequencies.

An excited hydrogen atom can emit photons of certain discrete frequencies in various regions of the electromagnetic spectrum. Three possible transitions are shown in Fig. 1.2.



An electron in the hydrogen atom is ionised by a photon of light with frequency 5.2×10^{16} Hz.

Calculate the kinetic energy of the ionised electron.

Ans:

Use Fig. 1.1 in (a) to identify the ionisation energy of the hydrogen atom:

- $E_i = 3.17 \times 10^{-17}$ J [1 mark]

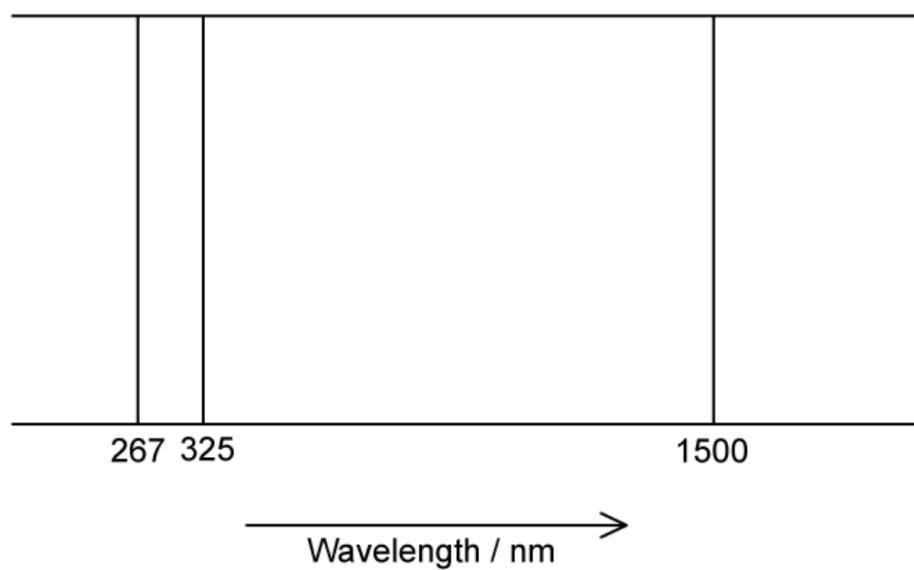
Use the photoelectric equation to calculate the energy absorbed from the photon:

- $E_p = hf$
- $E_p = (6.63 \times 10^{-34}) \times (5.2 \times 10^{16})$
- $E_p = 3.44 \times 10^{-17}$ J [1 mark]

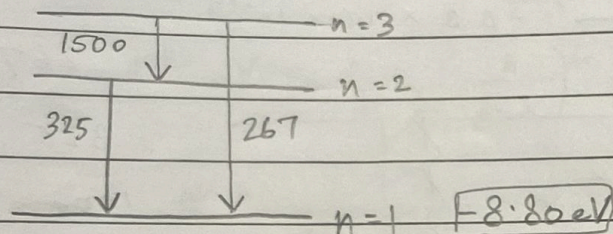
Calculate the kinetic energy of the electron:

- $KE = E_p - E_i$
- $KE = (3.44 \times 10^{-17}) - (3.17 \times 10^{-17})$
- $KE = 2.7 \times 10^{-18}$ J [1 mark]

An atom has a ground state energy of -8.80 eV . The complete line emission spectra for this atom is shown in Fig. 1.1.



3 wavelengths $\Rightarrow$ 3 transitions



$$\Delta E = \frac{hc}{\lambda}$$

$$E_3 - E_2 = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1500 \times 10^{-9}} = 0.83 \text{ eV}$$

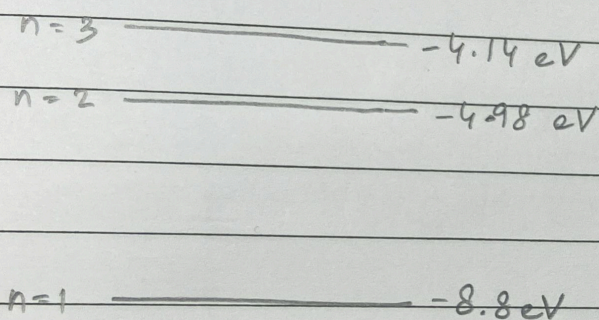
$$E_2 - E_1 = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{325 \times 10^{-9}} = 3.825 \text{ eV}$$

$$E_3 - E_1 = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{267 \times 10^{-9}} = 4.656 \text{ eV}$$

$$E_3 = 4.656 + (-8.8) = -4.14 \text{ eV}$$

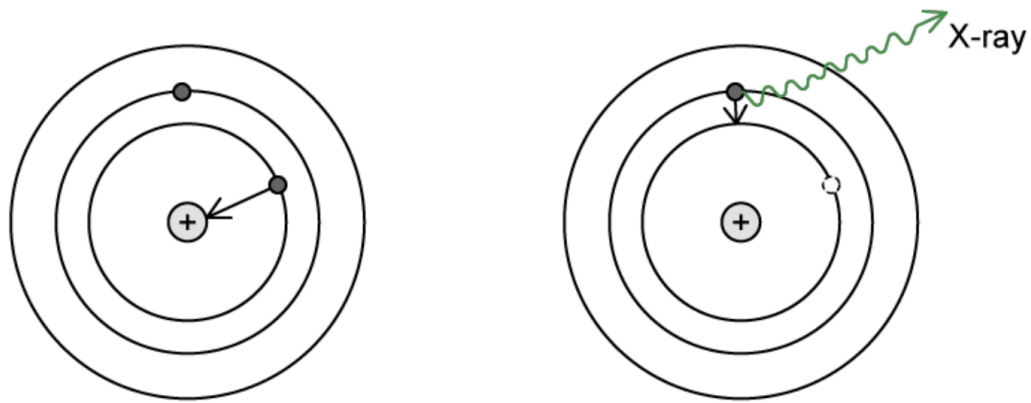
$$E_2 = 3.825 + (-8.8) = -4.98 \text{ eV}$$

$$E_1 =$$



Unstable nuclei can decay in a variety of ways.

Electron capture is one such process, during which the nucleus 'captures' an orbiting inner-shell electron. The resulting nucleus has one less proton and one more neutron. Sometimes this process is accompanied by the emission of a characteristic X-ray photon, as illustrated in Fig. 1.1.



Explain why an X-ray photon is produced in this process and suggest why it is called 'characteristic'.

- Electron in a higher energy state/ outer electron replaces the missing electron (which has been absorbed by the nucleus).
- Transition from higher to lower energy state releases an X-ray photon.
- X-ray photon is characteristic because its energy is equal to the difference in energy levels.

The Auger Effect is a process that sometimes accompanies that illustrated in Fig. 1.1. This process involves the emission of an electron – called an Auger electron – from the atom when a core electron is removed, leaving a vacancy. This is illustrated in Fig. 1.2.

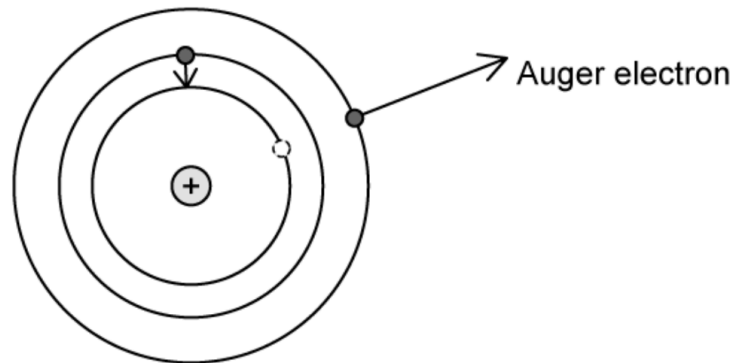


Fig. 1.2

This emission occurs if sufficient X-ray energy is transferred to outer shell electrons during the process.

Show that the speed of a typical Auger electron is around 22% of the speed of light, stating any assumptions you make in your calculation.

(Typical X-ray wavelength is $\sim 10^{-10}$ m).

Calculate the frequency of a typical X-ray photon:

- Wave speed, $c = f\lambda$
- $3 \times 10^8 = f \times (1 \times 10^{-10})$
- $f = 3 \times 10^{18} \text{ Hz}$ [1 mark]

Calculate the energy of a typical X-ray photon:

- $E = hf$
- $E = (6.63 \times 10^{-34}) \times 3 \times 10^{18} = 1.99 \times 10^{-15} \text{ J}$ [1 mark]

Calculate the speed of the Auger electron:

- If the energy of the photon is absorbed as kinetic energy of the Auger electron, then $E = hf = \frac{1}{2}mv^2$
- $1.99 \times 10^{-15} = 0.5 \times (9.11 \times 10^{-31}) \times v^2$
- $v = \sqrt{\frac{2 \times 1.99 \times 10^{-15}}{9.11 \times 10^{-31}}} = 6.6 \times 10^7 \text{ m s}^{-1}$ [1 mark]

Speed as a fraction of c:

- $\frac{6.6 \times 10^7}{3 \times 10^8} \times 100 = 22\% \text{ the speed of light;}$ [1 mark]

Assumption:

- This calculation assumes that all the energy from the X-ray photon is absorbed by the electron

OR

- There is a 1:1 interaction between photons and electrons; [1 mark]

When a beam of high-energy electrons is sent through a regularly spaced array of crystal atoms, they form a diffraction pattern of concentric bright and dark rings on a fluorescent screen, as shown in Fig. 1.3.

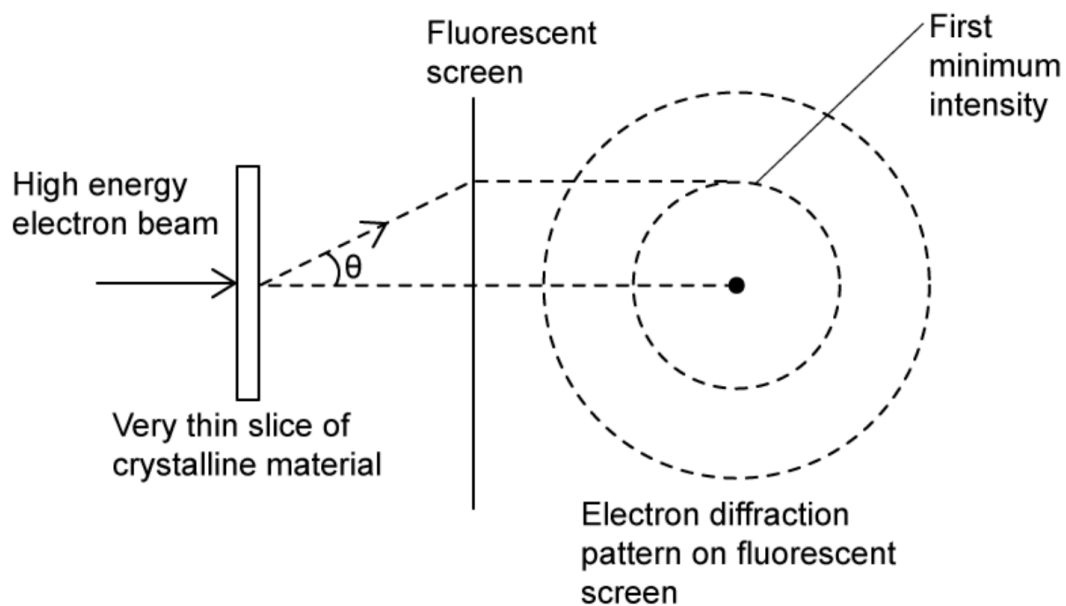


Fig. 1.3

It can be shown that the angle of electron-diffraction θ is related to the diameter of an atomic nucleus d by the equation

$$\sin \theta = 1.22 \frac{\lambda}{d}$$

Suggest what the term λ in the equation represents and describe how the observed diffraction pattern will vary in response to it.

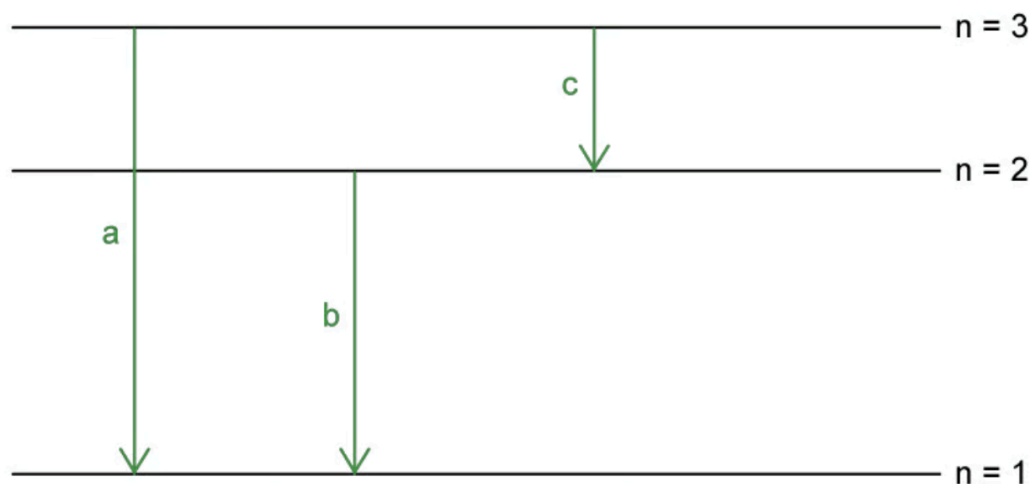
- λ = de Broglie wavelength of electrons
- As λ increases, $\sin \theta$ increases – which means diffraction angle θ increases (btwn 0-90°).
- As θ increases, diffraction pattern spreads out, and radius of concentric rings increases/ concentric rings get further apart.

Transitions between three energy levels in a particular atom give rise to three spectral lines. In decreasing magnitudes, these are f_1 , f_2 and f_3 .

Show that the equation which relates f_1 , f_2 and f_3 is:

$$f_1 = f_2 + f_3$$

Sketch a suitable diagram to illustrate your answer.



- Three energy levels shown with correct gap sizes; [1 mark]
- Three energy transitions labelled corresponding to three photons of decreasing energy (frequency); [1 mark]

Explanation of the energy level diagram and how it relates to spectral lines:

- The energy of transition (a) is the largest and the energy of transition (c) is the smallest
- Therefore (a) corresponds to a spectral line of frequency f_1 (the highest frequency); [1 mark]
- (b) corresponds to a spectral line of frequency f_2 **AND** (c) corresponds to a spectral line of frequency f_3 [1 mark]

Link photon energies to the spectral lines:

- The released energy in transition (a) must be equal to the released energy in (b) + the released energy in (c)
- This can be written as $hf_1 = hf_2 + hf_3$ **AND** therefore $f_1 = f_2 + f_3$ [1 mark]

Explain the significance of an electron at an energy level of 0eV.

- If an electron exists at energy level 0eV, it means the atom has become ionised.

- Enough energy has been transferred to the electron such that it is able to escape the atom.
- And is no longer bounded by the electrostatic force from the nucleus.

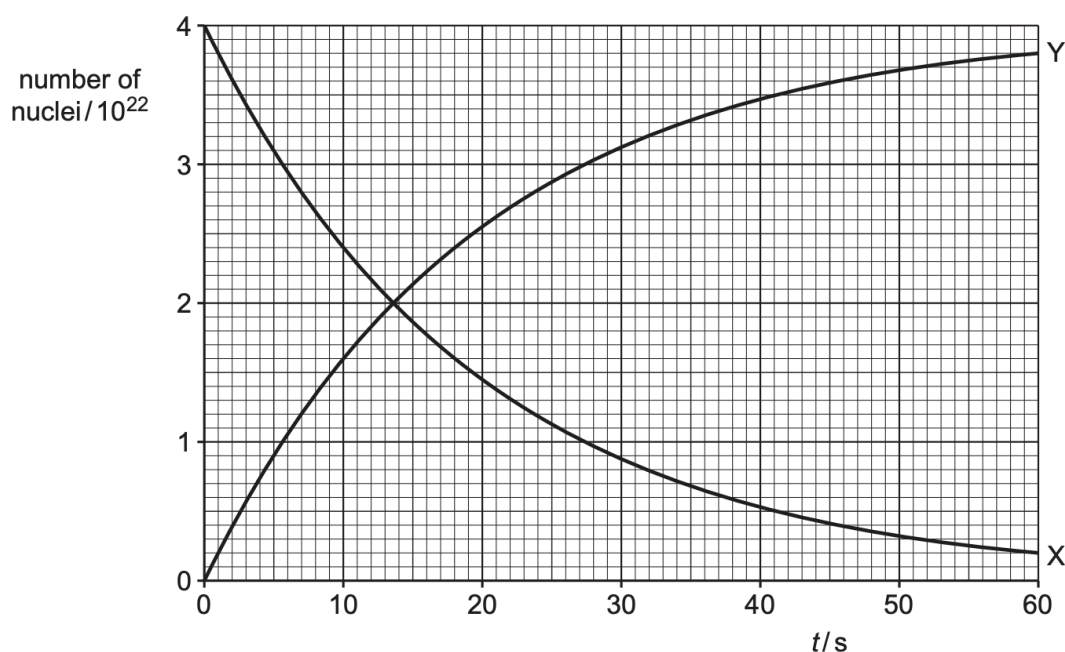
23. Nuclear physics

Define half-life of a radioactive isotope.

- time for activity (of sample) to halve.

(b) Radioactive isotope X decays to isotope Y.

A sample contains only nuclei of X at time $t = 0$. Fig. 9.1 shows the variation with t of the numbers of nuclei of X and of Y as the sample decays.



- State the name of the quantity represented by the magnitude of the gradient of line X in Fig. 9.1.**
 - Activity of X at time t
- State conclusions about X or Y that may be drawn from Fig. 9.1. The conclusions may be qualitative or quantitative. Use the space below for any working that you need.**
 - Y is a stable isotope
 - total number of nuclei is constant
 - half-life of X is 13.6 s
 - decay constant (of X) is 0.051 s^{-1}
 - amount of X at $t = 0$ is 0.066 mol
 - activity of X at $t = 0$ is $2.0 \times 10^{21} \text{ Bq}$

- c. The mass of radioactive isotope X in the sample in (b) is 7.3×10^{-4} kg at time $t = 0$. Determine the nucleon number of isotope X.

$$\text{mass of 1 nucleus} = (7.3 \times 10^{-4}) / (4.0 \times 10^{22})$$

$$\text{nucleon number} = \text{mass of nucleus} / (1.66 \times 10^{-27})$$

$$= (7.3 \times 10^{-4}) / (4.0 \times 10^{22} \times 1.66 \times 10^{-27})$$

$$= 11 \text{ and given as an integer}$$

24. Medical physics

State what is meant by a photon.

- packet / quantum of energy
- of electromagnetic radiation

Fig. 8.1 shows a tube in which X-rays are produced at a metal target.

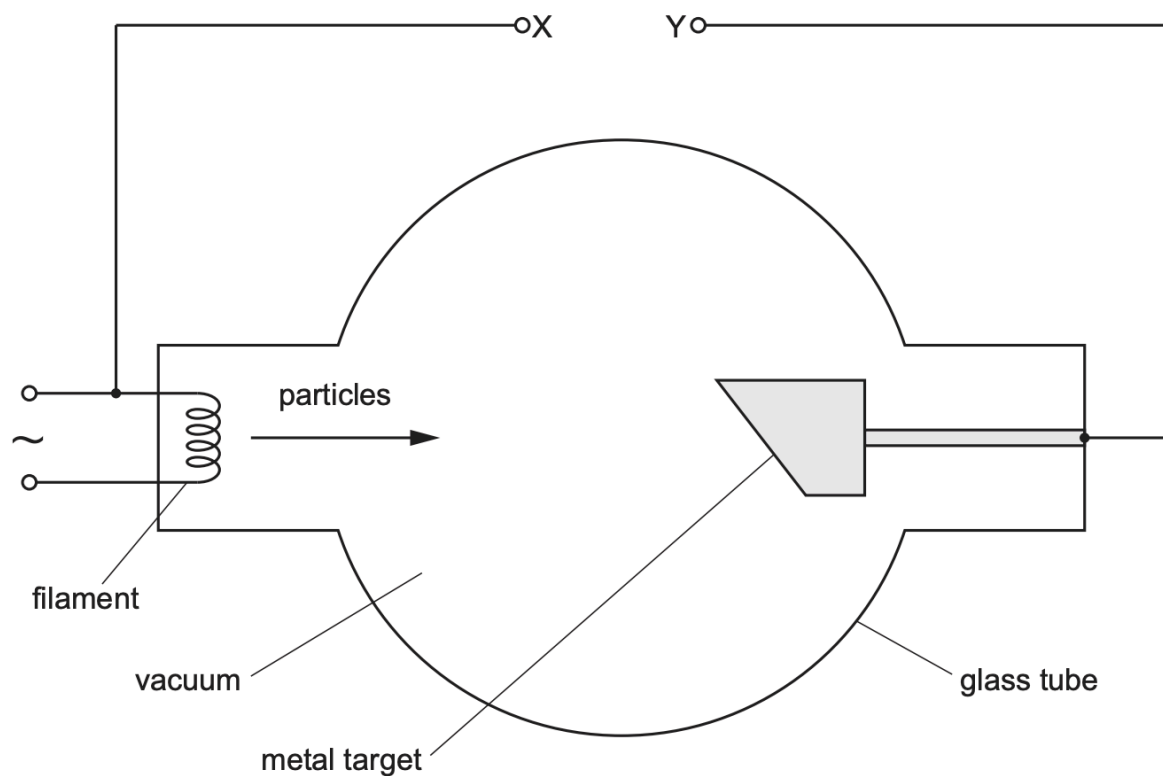


Fig. 8.1

X = negative terminal

Y = positive terminal

For an accelerating voltage of 32kV in Fig. 8.1, determine the maximum energy, in MeV, of an X-ray photon produced at the target.

- 0.032 MeV

NOTE: maximum energy of photon = eV. In this question, the unit is MeV and already has e so no need to multiply by e. To convert to joules, multiply by e and 10^6 .

NOTE: to calculate minimum wavelength of X-rays produced:

- E (energy of X-ray photon) = hf
- $\lambda = c/f$
- $\lambda = hc / E$

Explain why X-rays can be used to produce images of internal body structures that have good contrast.

- Bone has much higher density than soft tissue.
- Thus bone has much larger attenuation coefficient than soft tissue: in bone there is greater absorption, but lesser penetration and transmission of X-rays.
- Thus, transmitted intensities by bone and tissue are very different, leading to good contrast images.

Show that the momentum p of a photon of electromagnetic radiation with wavelength λ is given by

$$p = \frac{h}{\lambda}$$

where h is the Planck constant.

$p = E / c$
$E = hc / \lambda$ and completion of algebra leading to $p = h / \lambda$

A beam of red light of intensity 160 W m^{-2} is incident normally on a plane mirror, as shown in Fig. 8.1. The momentum of each photon in the beam is $9.5 \times 10^{-28} \text{ N s}$.

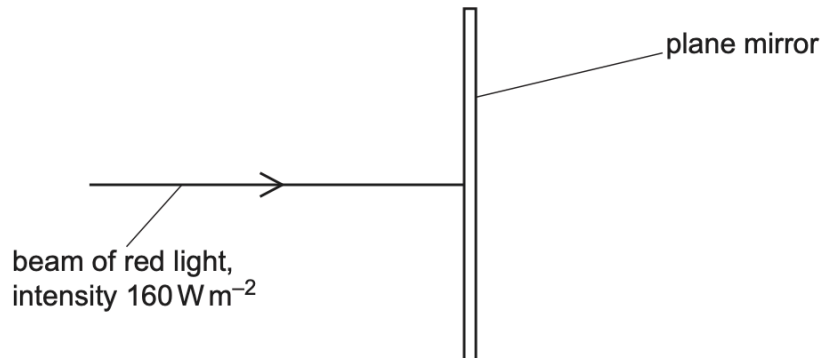


Fig. 8.1

All of the light is reflected by the mirror in the opposite direction to its original path. The cross-sectional area of the beam is $2.5 \times 10^{-6} \text{ m}^2$.

- a. Show that the number of photons incident on the mirror per unit time is $1.4 \times 10^{15} \text{ s}^{-1}$.

$$\begin{aligned}
 \text{Power} &= \text{Intensity} \times \text{area} \\
 &= 160 \times 2.5 \times 10^{-6} = 4 \times 10^{-4} \text{ J s}^{-1} \\
 \text{total momentum} &= \frac{E}{c} = \frac{4 \times 10^{-4}}{3 \times 10^8} = 1.33 \times 10^{-12} \\
 &\quad \text{(1 sec)} \\
 \text{no. of photons} &= \frac{1.33 \times 10^{-12}}{9.5 \times 10^{-28}} = 1.4 \times 10^{15} \\
 &\quad \text{(1 sec)}
 \end{aligned}$$

	Ultrasound	X-rays
method of production	vibrating quartz crystal	electrons hitting metal target
detected wave (absorbed, reflected or transmitted)	reflected	transmitted

Explain how X-rays are produced for use in medical diagnosis.

- electrons are accelerated (by an applied p.d.)
- electrons hit target

- X-rays produced when electrons decelerate

25. Astronomy and cosmology

Explain what is meant by a standard candle.

- An object of known luminosity

Explain how a standard candle can be used to measure astronomical distances.

- Standard candle has known luminosity.
- Measure the radiant flux intensity of the standard candle on earth.
- Use inverse square law of flux to calculate distance.
$$F = L/4\pi d^2$$

State 2 factors affecting radiant flux intensity from a star

- Luminosity of the star
- Distance from centre of star to earth

State 2 examples of standard candles

- Cepheid variable stars
- Type 1a supernovae

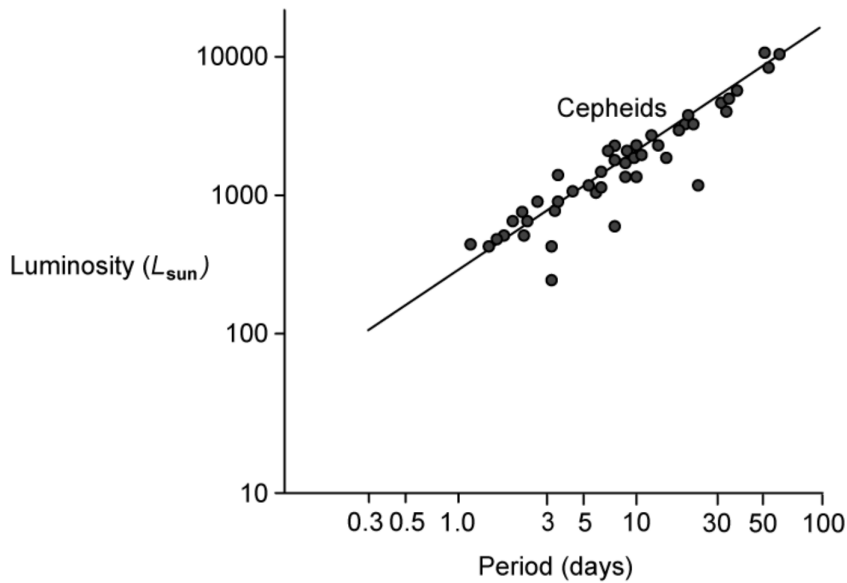
State what is meant by the luminosity of a star

- The total power of radiation emitted by the star

Suggest why type 1a supernovae are used to measure distances to the most distant galaxies.

- They have high luminosity, so are extremely bright.
- Their radiant flux intensity on earth will be detectable/ bright enough to measure even at very large distances.

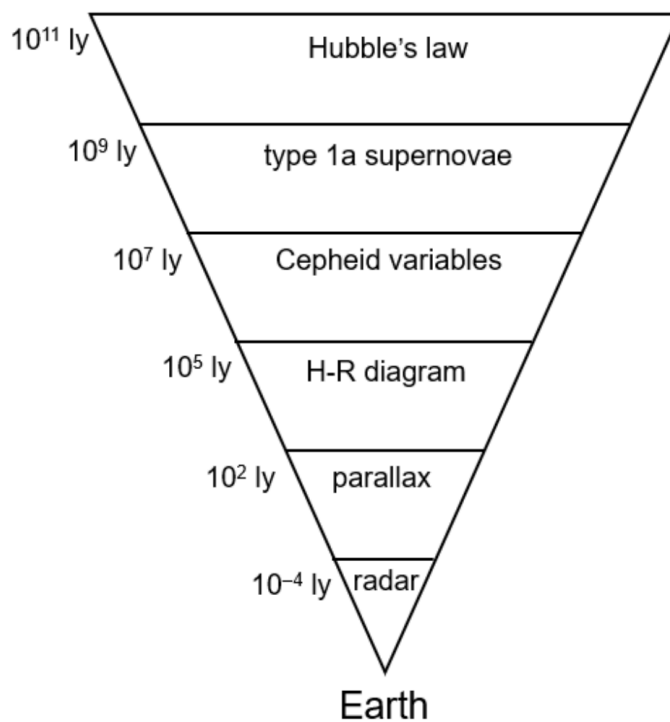
NOTE: billion = 10^9 .



Luminosity of the Sun = $3.8 \times 10^{26} \text{ W}$

NOTE: this graph is given with luminosity in terms of luminosity of sun on y-axis. So every value of y-axis x luminosity of sun = luminosity.

Fig. 1.1 shows a chart known as the cosmic distance ladder. It details different methods of measuring astronomical distances according to the distance they can measure from Earth.



The Andromeda galaxy is the closest galaxy to our galaxy (Milky Way). Suggest which method could be used to prove that Andromeda is a separate galaxy outside of Milky Way. Explain your choice.

- Cepheid variables
- Distances to nearby stars in Milky Way are less than 10^5 light years away OR Distances to other galaxies are more than 10^5 light years away.
- Andromeda is too close to Earth to use Hubble's law method.
- Andromeda is too far from Earth to use H-R diagram/ parallax method.
- Type 1a supernovae are rare events/ cannot be reliably predicted.

NOTE: size of Milky Way = 10^5 ly.

Recently, astronomers have observed a type 1a supernova called Encore in a distant galaxy. Measurements of the light from Encore revealed that the galaxy is 10 billion light-years away.

Light from supernova Encore is collected using the James Webb Space Telescope (JWST).

The JWST detects a peak power reading of 5.3×10^{-16} W. Theory predicts that this reading corresponds to Encore's peak luminosity of 2.4×10^{36} W.

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

(i) Calculate the peak radiant flux intensity at the JWST.

[3]

(ii) Show that the collecting area of the JWST is 25 m^2 .

[2]

Ans:

(i) Calculate the peak radiant flux intensity at the JWST:

Convert the distance from light-years to metres:

- Distance: $d = (10 \times 10^9) \times (9.46 \times 10^{15}) = 9.46 \times 10^{25} \text{ m}$ [1 mark]

Use the inverse square law of flux to calculate the peak intensity:

- Inverse square law of flux: $F = \frac{L}{4\pi d^2}$ [1 mark]

- Substitute: $F = \frac{2.4 \times 10^{36}}{4\pi(9.46 \times 10^{25})^2}$

- Peak radiant flux intensity: $F = 2.13 \times 10^{-17} \text{ W m}^{-2}$ [1 mark]

(ii) Show that the collecting area of the JWST is 25 m^2 :

- Intensity equation: $F = \frac{P}{A} \Rightarrow A = \frac{P}{F}$

- Substitute: $A = \frac{5.3 \times 10^{-16}}{2.13 \times 10^{-17}}$ [1 mark]

- Collecting area: $A = 24.9 = 25 \text{ m}^2$ [1 mark]

Solar panels consisting of combinations of photovoltaic cells use energy in the radiation received from the Sun to generate electricity.

An advertisement for solar panels claims that the intensity of radiation from the Sun incident at the top of the Earth's atmosphere is more than 2 kW m^{-2} .

Assess the validity of this claim.

Use the following data:

- radius of Sun = $6.96 \times 10^8 \text{ m}$
- surface temperature of Sun = 5790 K
- distance from Sun to Earth = $1.50 \times 10^{11} \text{ m}$

Ans:

Calculate the luminosity of the Sun:

- Stefan-Boltzmann law: $L = 4\pi r^2 \sigma T^4$
- $L = (5.67 \times 10^{-8}) \times 4\pi \times (6.96 \times 10^8)^2 \times (5790)^4$
- $L = 3.879 \times 10^{26} \text{ W}$ [1 mark]

Calculate the radiant flux intensity:

- Flux intensity: $F = \frac{L}{4\pi d^2}$
- $F = \frac{3.879 \times 10^{26}}{4\pi \times (1.50 \times 10^{11})^2} = 1371.92 \text{ W m}^{-2}$ [1 mark]
- $F = 1.37 \text{ kW m}^{-2}$ [1 mark]

Conclusion:

- $1.37 \text{ kW m}^{-2} < 2 \text{ kW m}^{-2}$ so the claim is false; [1 mark]

A space probe is sent on a mission from Earth to Mars.

Power for the space probe is provided by solar panels that collect radiant energy from the Sun. The solar panels have a total area of 31 m^2 and are directly facing the Sun throughout the journey. The efficiency of the solar panels is 30%.

The mean orbital radius of Mars about the Sun is $2.3 \times 10^{11} \text{ m}$.

Compare the power available to the probe when it leaves Earth's atmosphere to the power available on its arrival at Mars.

Ans:

Determine the power available to the probe at Earth:

- Power: $P = FA$
- The panels have an efficiency of 30%, so
- Total power available (at Earth): $P_E = 0.3 \times 1370 \times 31 = 12\,740\text{ W}$ [1 mark]

Determine the power available to the probe at Mars:

- Radiant flux intensity: $F = \frac{L}{4\pi d^2}$
- Since $F \propto \frac{1}{d^2}$ and $P \propto F \Rightarrow P \propto \frac{1}{d^2}$
- Ratio: $\frac{P_M}{P_E} = \left(\frac{d_E}{d_M}\right)^2 \Rightarrow P_M = P_E \times \left(\frac{d_E}{d_M}\right)^2$
- Total power available (at Mars): $P_M = 12\,740 \times \left(\frac{1.5 \times 10^{11}}{2.3 \times 10^{11}}\right)^2$ [1 mark]
- Total power available (at Mars): $P_M = 5420\text{ W}$ [1 mark]

Conclusion:

- The power available to the probe at Mars is 5420 W compared to 12 740 W at Earth, this is a decrease of 42.5%; [1 mark]

Neptune is the farthest known planet from the Sun in the Solar System. Its distance from the Sun is 30 times greater than the distance of the Earth from the Sun.

A similar space probe to the one in **(b)** is sent on a mission to Neptune.

(i) Calculate the area of solar panels that would be required to achieve the same availability of power at Neptune as the space probe in **(b)** when it was at Mars.

[3]

(ii) The International Space Station (ISS) currently holds the record for the largest array of solar panels in space, spanning a total area of 3244 m² to supply all of its power.

Using this information, and your answer to **(c)(i)**, comment on the current viability of the mission to Neptune.

[2]

Determine the radiant flux intensity at Neptune:

- Radiant flux intensity: $F = \frac{L}{4\pi d^2}$
- Ratio: $\frac{F_N}{F_E} = \left(\frac{d_E}{d_N}\right)^2 \Rightarrow F_N = F_E \times \left(\frac{d_E}{d_N}\right)^2$
- Radiant flux intensity at Neptune: $F_N = 1370 \times \left(\frac{d_E}{30d_E}\right)^2 = 1370 \times \left(\frac{1}{30}\right)^2$ [1 mark]
- Radiant flux intensity at Neptune: $F_N = 1.52 \text{ W m}^{-2}$ [1 mark]

Determine the required area of the panels:

- Power: $P = FA \Rightarrow A = \frac{P}{F}$
- Area of panels required: $A = \frac{5420}{1.52} = 3570 \text{ m}^2$ [1 mark]

(ii) Comment on the viability of the mission:

- Since $3570 \text{ m}^2 > 3244 \text{ m}^2$, this is not (currently) a viable mission; [1 mark]

Any one from:

- The ISS stays at a constant distance from the Sun whereas the probe will be moving away (from the Sun); [1 mark]
- The mass of the solar panels would make the launch (from Earth) difficult; [1 mark]
- A different energy source (e.g. nuclear power) would be more efficient / compact; [1 mark]
- The energy needs of the ISS and space probe are different; [1 mark]

State Wein's displacement law

- The wavelength of maximum intensity emitted by a star is inversely proportional to its thermodynamic temperature.
- $\lambda_{\text{max}} \propto 1/T$

The colour of a star indicates its surface temperature. In a sample of stars, the main colours observed are

white
blue
orange
yellow
red
blue-white
yellow-white

List the colours from the coolest to the hottest.

Ans:

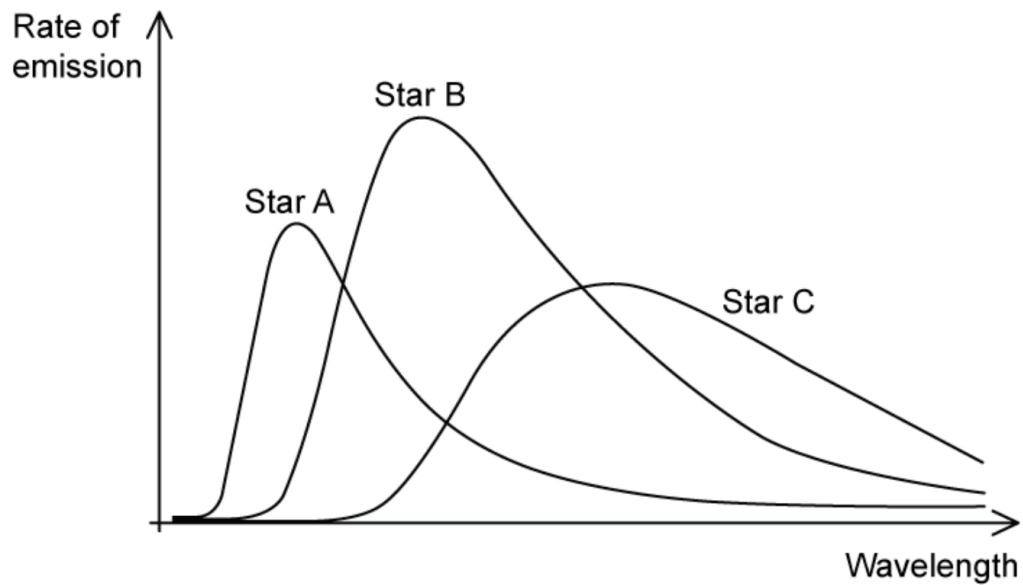
	colour
coolest	red
	orange
	yellow
	yellow-white
	white
	blue-white
hottest	blue

NOTE: imagine eating a hot iron rod.

dull red → orange → yellow → white → bluish-white

White appears before the hottest stage.

Fig. 1.1 shows how the rate of emission for three stars, A, B and C varies with wavelength.



Identify:

- Hottest star= A
- Coolest star= C
- Brightest star= B

State what factors the power output of a star depends on

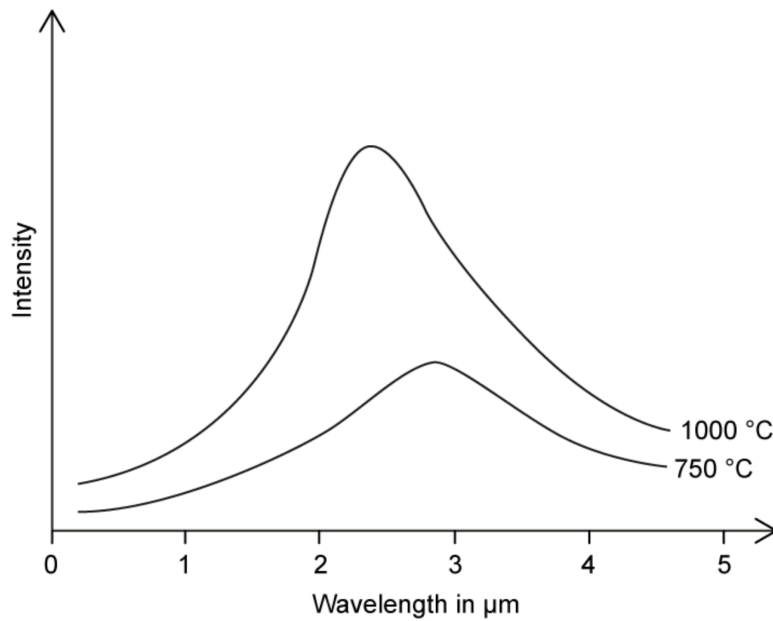
- Radius
- Surface temperature

In Table 1.1 below, place a tick (✓) next to the statement(s) that is/are associated with Wien's Law.

the lower the surface temperature of a star, the shorter the wavelength it emits at the peak intensity	
the relationship between the highest intensity wavelength and surface temperature of a star is inversely proportional	
the higher the surface temperature of a star, the greater the intensity of the radiation emitted at each wavelength	
the relationship between the lowest intensity wavelength and surface temperature of a star is inversely proportional	

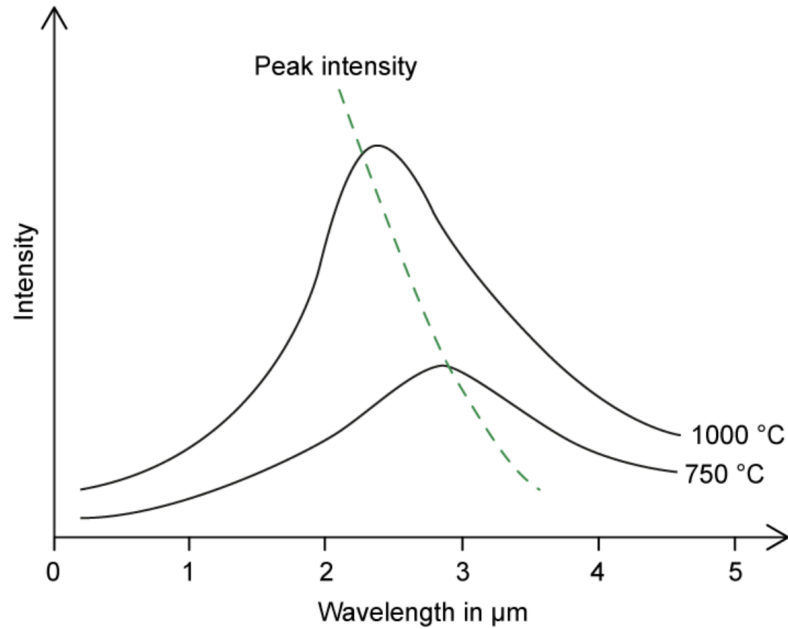
Ans: B and C

Fig. 1.1 shows the black body radiation curves of two stars with surface temperatures 750°C and 1000°C respectively.



On Fig. 1.1, sketch a curve to show how the peak intensity of radiation varies with wavelength.

Ans:



The radius of the star is 5.6×10^9 m.

The Stefan-Boltzmann law can be written as:

$$L = \sigma AT^4$$

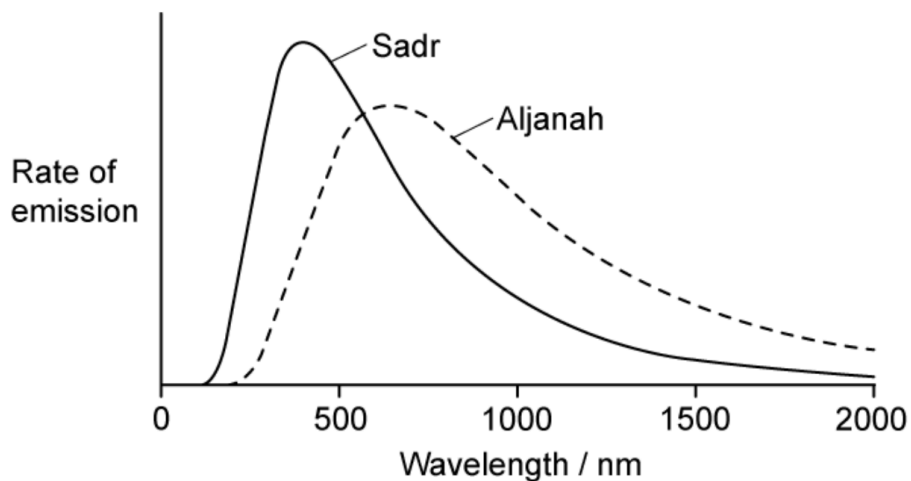
(i) State the meaning of L , A and T .

[3]

Ans:

- L = luminosity / total power output (of the star); [1 mark]
- A = surface area (of the star); [1 mark]
- T = surface temperature (of the star); [1 mark]

Fig. 1.2 shows the rate of emission from two of the stars, Sadr and Aljanah, plotted against wavelength.



Explain how the curves in Fig. 1.2 can be used to determine the surface temperature of the stars.

Ans:

- Wein's law states that the wavelength of maximum intensity emitted by a star is inversely proportional to its thermodynamic temperature.

$$\lambda_{\text{max}} \propto 1/T$$

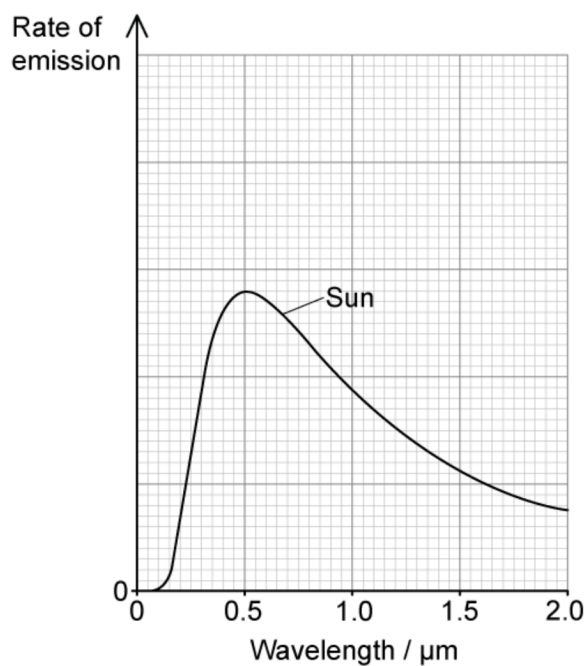
- Read off λ_{max} from the graph.
- Use a reference star/ star with known λ_{max} and T to find a constant of proportionality.

In a comparison of two stars, A and B, the following data was collected

Surface temperature of star A = 25 000 K

Surface temperature of star B = 4300 K

Fig. 1.1 shows the variation of rate of emission against wavelength for the Sun.

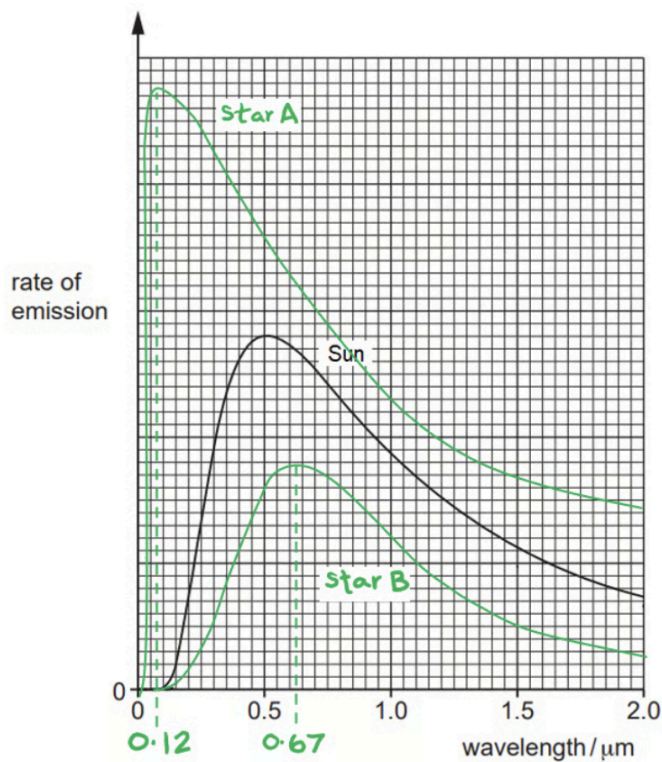


On Fig. 1.1, sketch the variation of rate of emission against wavelength for stars A and B.

Maximum wavelength of star A: $\lambda_A = 1.154 \times 10^{-7} \text{ m} = 0.1154 \mu\text{m}$ (from (c))

Maximum wavelength of star B: $\lambda_B = 6.709 \times 10^{-7} \text{ m} = 0.67 \mu\text{m}$ (from (c))

Ans:



In recent years, astronomers have discovered sources of fast radio bursts (FRBs) in other galaxies. Studies suggest that these sources may be a type of standard candle.

An FRB source emits intense bursts of radio waves, each burst lasting for a fraction of a second. The closest FRB source is in a massive spiral galaxy 4.60×10^{24} m from the Earth. A detector of area 1.00×10^{-4} m² on the surface of the Earth received bursts of radio waves. In one burst, 9.40×10^{-23} J of energy was received in a time of 1.15 ms.

(i) Show that the luminosity of the source is about 2×10^{35} W.

[4]

(ii) When FRB sources were first discovered, some observers suggested that the bursts might be extraterrestrial communications.

Suggest why this is unlikely.

luminosity of the Sun = 3.8×10^{26} W

[2]

Determine the power of the burst of radio waves:

- Power: $P = \frac{E}{t} = \frac{9.40 \times 10^{-23}}{1.15 \times 10^{-3}} = 8.17 \times 10^{-20} \text{ W}$ [1 mark]

Determine the intensity of the burst:

- Intensity: $F = \frac{P}{A} = \frac{8.17 \times 10^{-20}}{1.00 \times 10^{-4}} = 8.17 \times 10^{-16} \text{ W m}^{-2}$ [1 mark]

Use the inverse square law of flux equation to find luminosity:

- Inverse square law of flux: $F = \frac{L}{4\pi d^2}$
- $L = 4\pi d^2 F = 4\pi \times (4.60 \times 10^{24})^2 \times (8.17 \times 10^{-16})$ [1 mark]
- $L = 2.17 \times 10^{35} = 2.2 \times 10^{35} \text{ W}$ [1 mark]

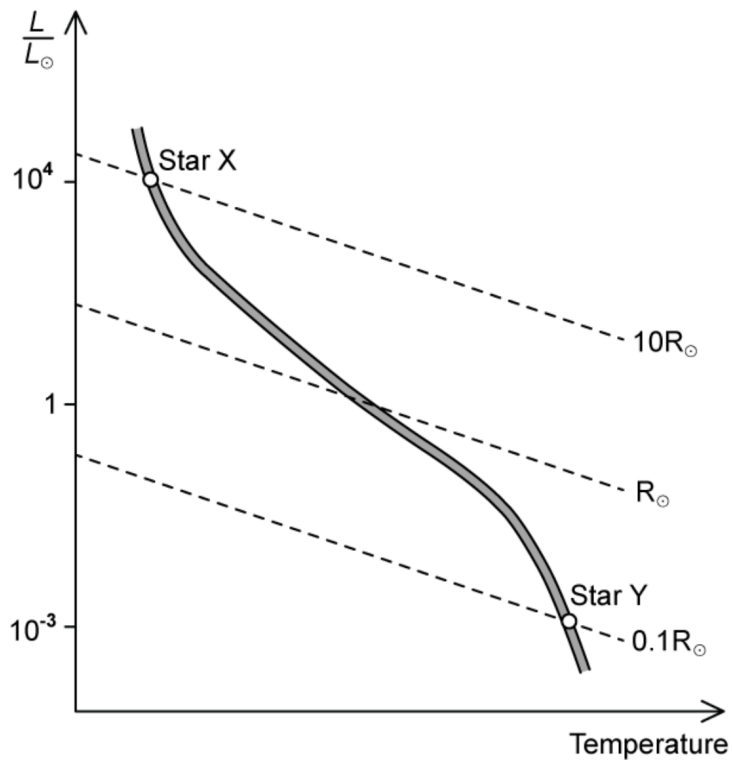
(ii) It is unlikely that FRB sources are extraterrestrial communications because:

- The luminosity of the FRB source is much larger than the luminosity of the Sun; [1 mark]
- A power output this large is unlikely to be artificially produced; [1 mark]

NOTE: a reasonable estimate for the diameter of a big city = 10–100 km.

Fig. 1.1 shows the logarithmic relationship between luminosity $\frac{L}{L_{\odot}}$ and temperature $\frac{T}{T_{\odot}}$ for a sample of stars, where $L_{\odot}$ and $T_{\odot}$ represent the luminosity and temperature of the Sun respectively.

Two stars, X and Y, are shown with their corresponding radii, in terms of the radius of the Sun $R_{\odot}$.



Deduce the colours of star X and star Y using the information in Fig. 1.1 and Table 1.1.

Table 1.1

star colour	surface temperature / $T_{\odot}$
blue	> 5.5
blue-white	$1.7 - 5.5$
white	$1.3 - 1.7$
yellow-white	$1.0 - 1.3$
yellow	$0.8 - 1.0$
orange	$0.6 - 0.8$
red	< 0.6

$$L = 4\pi r^2 \sigma T^4$$

$$L_{\odot} = 4\pi R_{\odot}^2 \sigma T_{\odot}^4$$

$$\Rightarrow T_{\odot} = \sqrt[4]{\frac{L_{\odot}}{4\pi R_{\odot}^2 \sigma}} \rightarrow \text{for sun}$$

$$\text{For X, } T = \sqrt[4]{\frac{10^4 L_{\odot}}{4\pi (10R_{\odot})^2 \sigma}} = T_{\odot} \times \sqrt[4]{\frac{10^4}{10^2}} = 3.16 T_{\odot} \quad \therefore \text{blue-white}$$

$$\text{For Y, } T = \sqrt[4]{\frac{10^{-3} L_{\odot}}{4\pi (0.1R_{\odot})^2 \sigma}} = T_{\odot} \times \sqrt[4]{\frac{10^{-3}}{(0.1)^2}} = 0.56 T_{\odot} \quad \therefore \text{red}$$

In 2017, an ultra-cool star in our galaxy Trappist-1 was discovered with at least five of its own orbiting planets.

Astronomers have studied it extensively to determine if there is a possibility of finding life on some of the planets orbiting Trappist-1.

Table 1.1 below shows some data.

Table 1.1

	Trappist-1	Sun
Luminosity $L/L_{\odot}$	5.26×10^{-4}	1
Surface temperature $T/T_{\odot}$	0.4	1
Radius of star $/R_{\odot}$	R	1
Distance between Earth and Sun / m	–	1.5×10^{11}
Distance between planets and Trappist-1 / m	1.6×10^9 to 9.0×10^9	–

The temperature T , in K, of a planet, its distance d from the star and the luminosity L of the star are related by the expression

$$\frac{T^4 d^2}{L} = 419 \times 10^3$$

(ii) The average temperature of the Earth is about 290 K.

Using data from Table 1.1, comment on the possibility of life existing on some of the planets orbiting Trappist-1.

[4]

Handwritten calculations and reasoning:

$$L_0 = \frac{T^4 d^2}{419 \times 10^3} = 3.798 \times 10^{26} \quad (\text{of sun})$$

range of temperatures on trappist:

$$T = \sqrt[4]{\frac{419 \times 10^3 \times 3.798 \times 10^{26} \times 5.26 \times 10^{-4}}{d^2}}$$

180 K - 425 K

→ 290 K lies btwn range, so it is possible for life to exist.

→ although surface temp. ^{of trappist} < sun, planets can be similar temp as earth, as planets orbiting trappist are closer to it.

However, for life to exist, temp is not the only factor

State hubble's law

- Recessional speed of a galaxy is directly proportional to its distance from the earth. OR

$$H_0 = \frac{v}{d}, \text{ where } v = \text{speed of recession of galaxy (from observer), } d = \text{distance of galaxy (away from observer); [1 mark]}$$

Explain how hubble's constant can be used to determine an age for the universe.

- $H_0 = v/d$, so H_0 has units of s^{-1}
Age of universe = $1/H_0$

Statement	Correct (✓)
The universe began 14 million years ago	
There was a giant explosion known as the big bang	✓
This caused the universe to expand from a single point	✓
It heated as it expanded to form the universe today	
Each point in the universe expands away from the others	✓
So, the further away galaxies are the slower they are moving	

- One mark for each correct tick

[Total: 3 marks]

- The universe began 14 **billion** and not 14 million years ago
- The universe has **cooled** and not heated as it has expanded
- The further away galaxies are, the **faster** they are moving

State what is meant by redshift

- Apparent wavelength of light is observed to be greater than known/ measured value
- Due to object/ star/ galaxy moving away from observer.

Explain how to determine that light from distance star has undergone redshift.

- Examining the lines in the spectrum of light from the star.
- And comparing the lines with known spectrum of light measured on earth.
- Redshift has occurred if the wavelengths are greater than measured values on earth.
- This redshift shows that the stars in distant galaxies are moving away from earth.

Describe the main ideas of the big bang theory.

- The universe started with an explosion/ the big bang
- All matter and energy was concentrated at a single point.
- The explosion caused the universe to expand from this single point.

Explain why Hubble's law and the expanding universe lead to Big Bang theory of origin of universe.

- All parts of the universe are moving away from each other.
- More distant objects are moving away faster.
- Thus matter must have been close together/ very dense in the past.

Explain why the redshift of galaxies is considered to be evidence for the big bang theory.

- The more distant the galaxy, the longer the light travels through space and expands/ stretches OR the greater the redshift OR the faster it recedes away from observer.
- Increasing redshift suggests that universe is expanding.
- If the universe is expanding, then it must have been closer together in the past.
- The expansion of the universe can be extrapolated back to a single point of origin (known as the Big Bang)

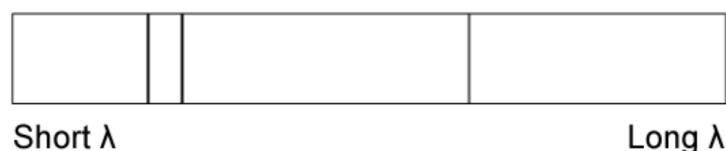
State the significance of the term H_0

- Age of the universe = $1/H_0$

Distance from the earth is known for many type 1a supernovae. Describe how these distances are used, with other data, to determine Hubble constant.

- Other data: recessional velocity
- Obtained from measurements of the wavelength of spectral lines (from type 1a supernovae)
- Plot a graph of velocity against distance
- Hubble constant is the gradient of the graph

Fig. 1.1 shows the emission spectrum of some wavelengths of light that were observed using a source in a laboratory.



The same wavelengths of light were observed on an emission spectrum from a distant galaxy but were found to be redshifted.

Describe how you would expect the emission spectrum of the distant galaxy to look compared to the spectrum in Fig. 1.1.

Ans:

- All the lines moved/ shifted towards the long λ side
- By the same amount

An astronomer observes that light from a galaxy has been shifted towards the blue end of an emission spectrum.

State and explain what can be deduced about the motion of the galaxy from this observation.

Ans:

- Galaxy is moving towards the observer
- Blueshift is a decrease in observed wavelength of light emitted from incoming stars/ galaxies OR Light from distant galaxies has a shorter wavelength than the same light observed on earth.

Hydrogen is an element of particular interest to cosmologists, as it can emit visible light waves.

A lot of useful information can be obtained by measuring the wavelength of the emitted light from the hydrogen in distant galaxies and comparing them to a laboratory sample. Some sample data is shown in Table 1.2.

Table 1.2

	Wavelength in nm
Laboratory sample	486.14
Galaxy 1a	486.40
Galaxy 1b	484.85

Galaxy 1a and galaxy 1b are both moving relative to Earth.

Compare the motions of galaxy 1a and galaxy 1b relative to Earth.

Ans:

Determine the change in wavelength for galaxy 1a:

- $\Delta\lambda = \lambda - \lambda_0$
- $\Delta\lambda_{1a} = 486.40 - 486.14 = 0.26 \text{ nm}$
- (Therefore,) galaxy 1a is moving away from Earth; [1 mark]

Determine the change in wavelength for galaxy 1b:

- $\Delta\lambda_{1b} = 484.85 - 486.14 = -1.29 \text{ nm}$
- (Therefore,) galaxy 1b is moving towards Earth; [1 mark]

State their relative speeds:

- (And) galaxy 1b is moving faster than galaxy 1a; [1 mark]

[Total: 3 marks]

If $\Delta\lambda$ is **positive** then the galaxy is **moving away** from the Earth. This is also known as **redshift**.

If $\Delta\lambda$ is **negative** then the galaxy is **moving towards** the Earth. This is also known as **blueshift**.

Use data from Table 1.2 to calculate the speed of galaxy 1b relative to Earth.

Ans:

- Wavelength in laboratory, $\lambda_0 = 486.14 \text{ nm}$
- Wavelength from galaxy 1b, $\lambda = 484.85 \text{ nm}$
- Speed of light, $c = 3.0 \times 10^8 \text{ m s}^{-1}$
- Change in wavelength for galaxy 1b, $\Delta\lambda = (-)1.29 \text{ nm}$ (from **(c)**)

Rearrange the equation and substitute values, to calculate v :

- $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c} \Rightarrow v = c \times \frac{\Delta\lambda}{\lambda_0}$ [1 mark]
- $v = (3.0 \times 10^8) \times \frac{1.29}{486.14}$ [1 mark]
- Speed of galaxy 1b = 796 067 = $7.96 \times 10^5 \text{ m s}^{-1}$ [1 mark]

Cosmologists commonly quote the recessional velocity v of astronomical objects in relation to the speed of light c . This ratio is known as redshift z :

$$z = \frac{v}{c}$$

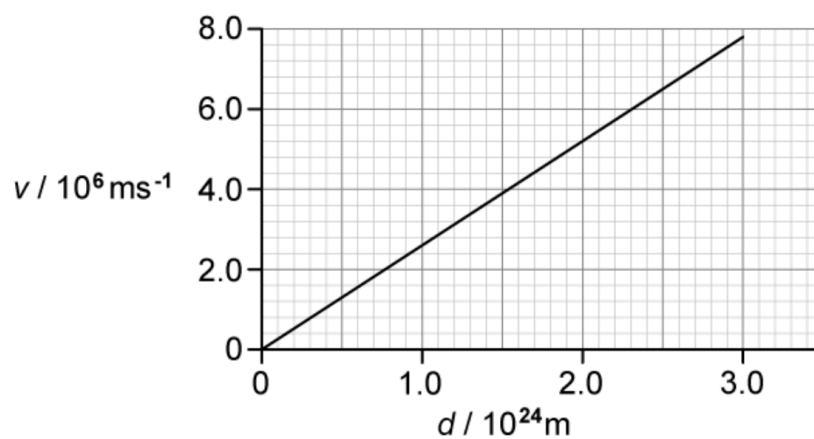
The Andromeda galaxy is the closest galaxy to our own Milky Way. Andromeda has a redshift value of $z = -0.001$.

State the significance of the minus sign and discuss its implication for the Milky Way and Andromeda in the distant future.

Ans:

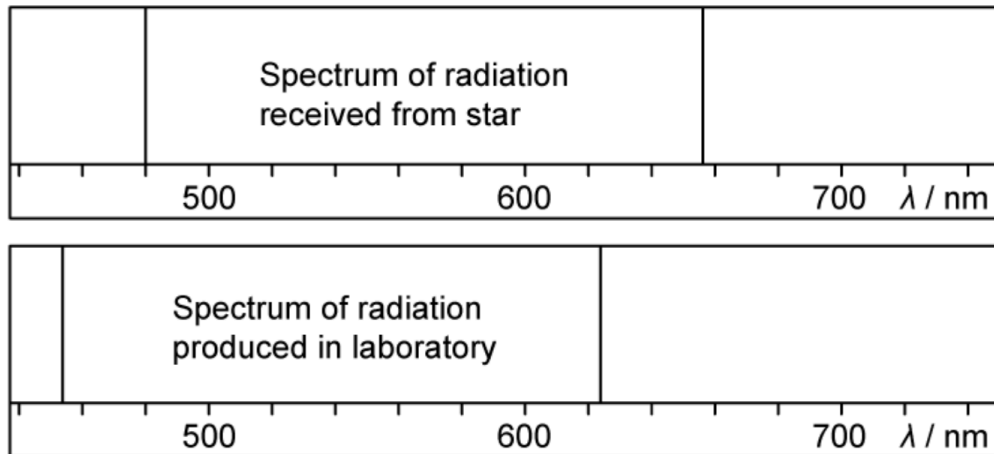
- Light from Andromeda is bluishshifted/ it is moving towards Milky Way.
- Andromeda will hit/ collide with Milky Way in future.

Fig. 1.1a shows the variation of recession velocity v for a number of galaxies with their approximate distance d from Earth.



Radiation from a star in a distant galaxy is analysed.

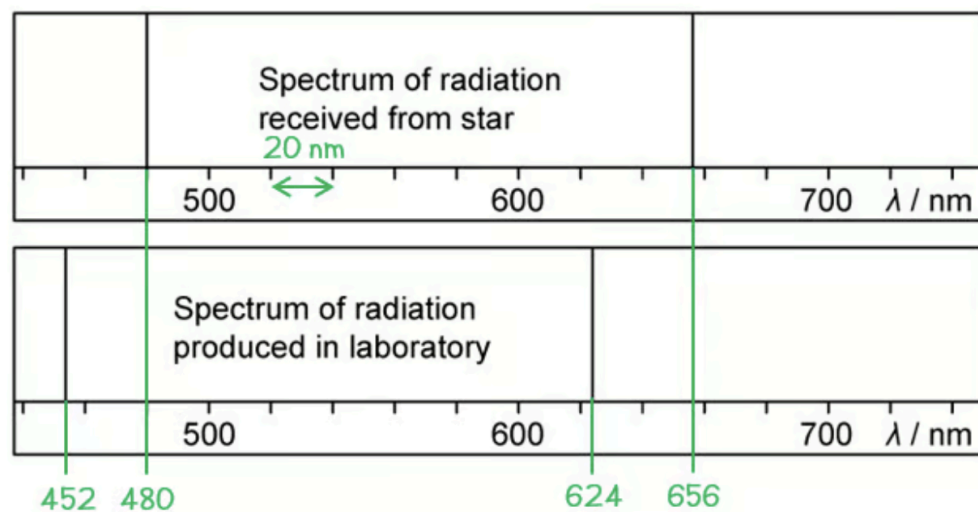
Fig. 1.1b shows the spectral lines of the radiation from the star at different wavelengths compared to the same spectral lines produced by a source in the laboratory.



Using the information in Fig. 1.1a and 1.1b, determine the distance of this galaxy from Earth.

Ans:

Use the line spectra to identify the wavelengths of the emitted radiation:

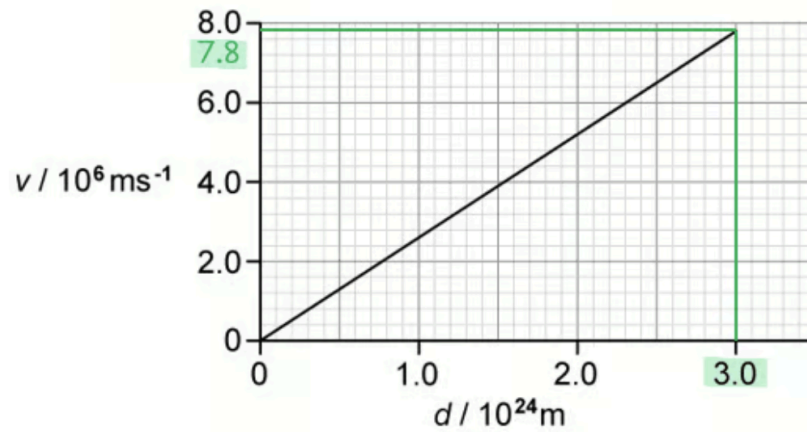


- (From diagram) $\lambda_{\text{star}} = 480 \text{ nm}$ **AND** $\lambda_{\text{lab}} = 454 \text{ nm}$ [1 mark]
- Shift in wavelength: $\Delta\lambda = 480 - 454 = 26 \text{ nm}$ [1 mark]
 Acceptable range for $\lambda_{\text{star}} = 479 \text{ to } 480 \text{ nm}$
 Acceptable range for $\lambda_{\text{lab}} = 452 \text{ to } 456 \text{ nm}$

Determine the recessional velocity of the galaxy:

- Doppler redshift: $\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c} \Rightarrow v = c \times \frac{\Delta\lambda}{\lambda}$
- $v = (3 \times 10^8) \times \frac{26}{454} = 1.72 \times 10^7 \text{ m s}^{-1}$ [1 mark]

Determine the gradient (Hubble constant) from the graph



- Hubble's law: $v = H_0 d$
- $H_0 = \frac{\Delta v}{\Delta d} = \frac{7.8 \times 10^6}{3.0 \times 10^{24}} = 2.6 \times 10^{-18} \text{ s}^{-1}$ [1 mark]

Determine the distance to the galaxy:

- $d = \frac{v}{H_0} = \frac{1.72 \times 10^7}{2.6 \times 10^{-18}} = 6.6 \times 10^{24} \text{ m}$ [1 mark]

Astronomers use the cosmic scale factor R to describe the scaling up of physical distances in the Universe.

As the universe expands, the wavelength of the detected radiation at a time t is larger than it was at a time t_0 in the past by a factor of

$$\frac{R}{R_0} = 1 + z$$

This expression represents the expansion rate of the universe.

Where R represents the separation of two galaxies at time t , R_0 represents the value of R at the time t_0 the radiation was emitted, and z represents the redshift, which is given by

$$z \approx \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

(i) Using the scale factor, or otherwise, determine, in years, the approximate age of the universe at the instant when the detected light from the distant galaxy in **(a)** was emitted.

[4]

(ii) State the assumption that you made in your answer to **(b)(i)**.

[1]

Ans:

$$\begin{aligned} \frac{R}{R_0} &= 1 + z \\ z &= \frac{\Delta\lambda}{\lambda} = \frac{480 - 454}{454} = \frac{13}{227} \\ \frac{R}{R_0} &= \frac{t}{t_0} = 1 + \frac{13}{227} = \frac{240}{227} \\ \text{let } t_0 &= \text{age till when light was emitted} \\ t &= \text{present age} = 1/H_0 = 1/2.6 \times 10^{-18} = 3.846 \times 10^{17} \\ &\Rightarrow \frac{3.846 \times 10^{17}}{60 \times 60 \times 24 \times 365} = 1.22 \times 10^{10} \text{ yrs} \\ \Rightarrow t_0 &= \frac{1.22 \times 10^{10}}{\frac{240}{227}} = 1.15 \times 10^{10} = 11.5 \text{ billion yrs old} \\ &\quad \text{universe was when the light was emitted.} \end{aligned}$$

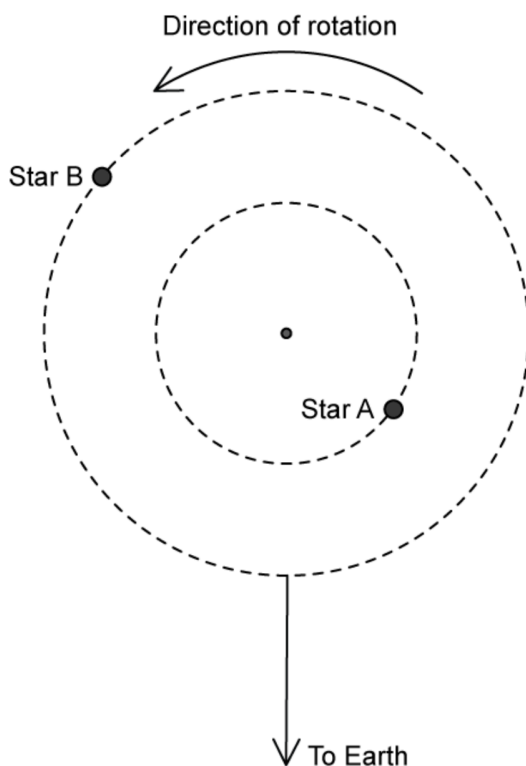
Assumption:

- Rate of expansion of universe is constant/ uniform/ not accelerated OR H_0 is constant over time.

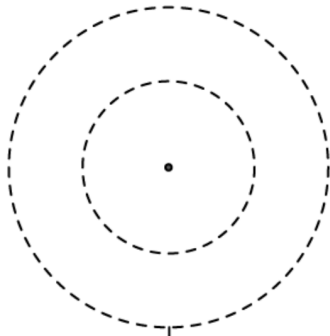
Discuss why different measurements of Hubble constant do not agree with each other.

- Measurements from distant galaxies have large uncertainties.
- Galaxies that are close by may have peculiar motion, eg. moving towards us/ orbiting around a centre of mass.
- Galaxies were not formed at the beginning of the universe, so H_0 is a lower limit.
- Rate of expansion may not be constant/ may be accelerating.

Two stars, A and B, in a binary system move in an anti-clockwise direction around a common centre of mass, as shown in Fig. 1.1.



On Fig. 1.2a and Fig. 1.2b below, mark the positions of stars A and B corresponding to their spectra as observed on Earth.

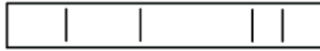


↓ To Earth

λ increasing →



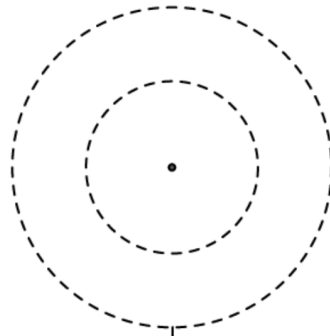
Lab sample



Star A



Star B



↓ To Earth

λ increasing →



Lab sample

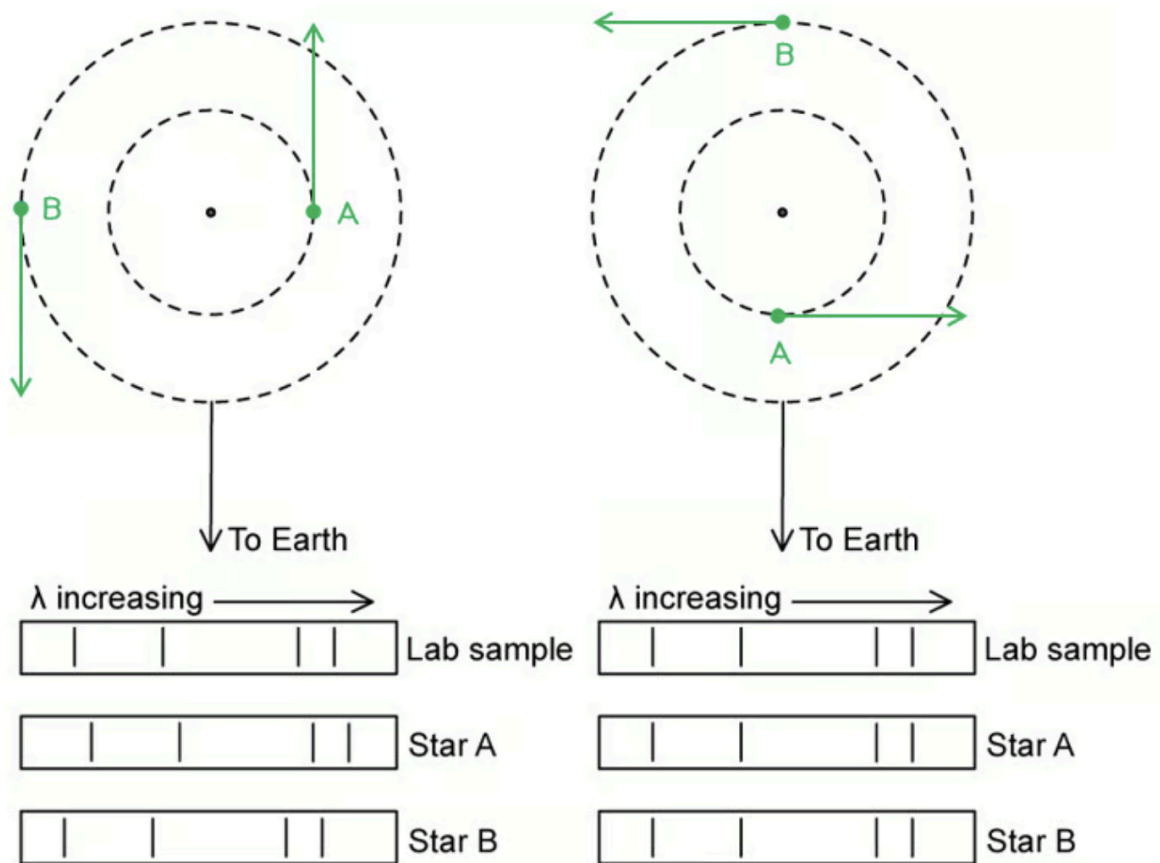


Star A



Star B

Ans:



On Fig. 1.2a...

- Star A (inner orbit) is marked to the right of the centre of mass with an arrow directed away from the Earth; [1 mark]
- Star B (outer orbit) is marked to the left of the centre of mass with an arrow directed towards the Earth; [1 mark]

On Fig. 1.2b...

- Both stars are marked directly above/below the centre of mass (in their respective orbits); [1 mark]

Newton assumed that the universe was infinite, uniform and static. The Big Bang model suggests space and time originated at one point around 14 billion years ago. At this time the temperature was very high.

In 1965, Penzias and Wilson discovered cosmic microwave background (CMB) radiation with a wavelength of around 1 mm.

Determine the current temperature of the CMB.

Surface temperature of the Sun = 5800 K

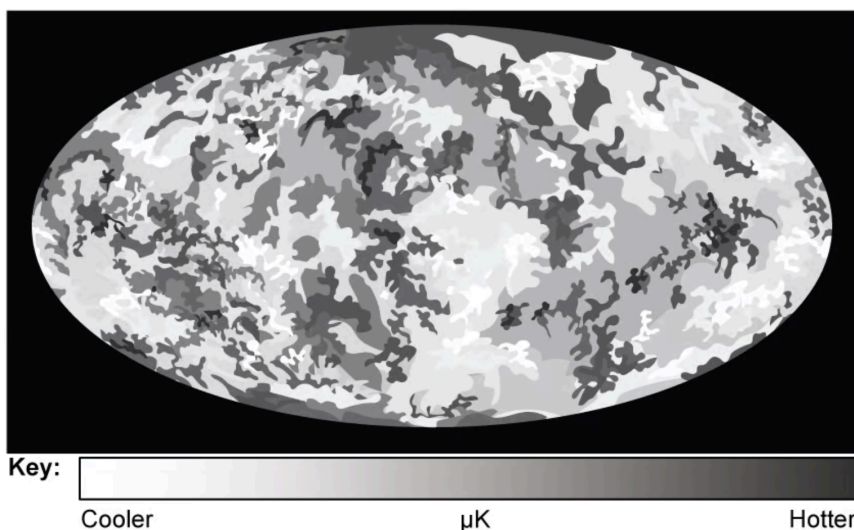
Peak wavelength emission of the Sun = 500 nm

Ans:

Use Wien's law to determine the temperature of the CMB:

- Wien's law: $T \propto \frac{1}{\lambda_{max}}$
- Therefore, $\frac{T_{CMB}}{T_{Sun}} = \frac{\lambda_{Sun}}{\lambda_{CMB}} \Rightarrow T_{CMB} = T_{Sun} \times \frac{\lambda_{Sun}}{\lambda_{CMB}}$
- $T_{CMB} = 5800 \times \frac{500 \times 10^{-9}}{1 \times 10^{-3}}$ [1 mark]
- $T_{CMB} = 2.9 \text{ K}$ [1 mark]

Fig. 1.1 shows the map of microwave background radiation from the COBE satellite. It shows the variation in the mean temperature of the CMB in different parts of the sky.



This radiation was first emitted around 300 000 years after the Big Bang, during a stage of the universe's history called the 'era of recombination'.

At this stage, the radiation was in the visible region of the spectrum and had a temperature of around 3000 K.

Determine the factor by which the universe has expanded since the universe was at a temperature of 3000 K.

- List the known quantities:
 - Initial temperature, $T_{then} = 3000 \text{ K}$
 - Final temperature, $T_{now} = 2.9 \text{ K}$ (from **(a)**)

Use the relationship between energy and temperature for a photon:

- Energy-temperature relation for photons: $E \propto T$
- $\frac{\text{energy of photons at 3 K}}{\text{energy of photons at 3000 K}} = \frac{kT_{now}}{kT_{then}} = \frac{2.9}{3000}$ [1 mark]
- $\frac{\text{energy of photons at 3 K}}{\text{energy of photons at 3000 K}} = 0.00097$ [1 mark]

Use the equation for photon energy

- Photon energy: $E = \frac{hc}{\lambda} \Rightarrow \lambda \propto \frac{1}{E}$
- $\frac{\lambda \text{ of photons at 3 K}}{\lambda \text{ of photons at 3000 K}} = \frac{1}{0.00097} = 1030$ [1 mark]

Using Fig. 1.1, your answers to **(a)** and **(b)**, and your knowledge of redshift and cosmology

(i) Describe how the CMB provides evidence for the Hot Big Bang model of the universe.

[2]

(ii) State and explain what the COBE data in Fig. 1.1 suggests about the temperature variation in the early universe.

[2]

(i) The CMB provides evidence for the Hot Big Bang model of the universe because:

EITHER

- The wavelength of the CMB has been redshifted by an expanding and cooling universe; [1 mark]
- The shorter wavelength in the past indicates the temperature must have been very high at the beginning; [1 mark]

OR

- The temperature of the CMB has cooled considerably from $t = 300\,000$ to now and even more since the Big Bang; [1 mark]
- This cooling was caused by the expansion of the universe / the stretching of spacetime; [1 mark]

(ii) The COBE data suggests that:

- The wavelength / intensity of the CMB is (almost) the same in all directions **OR** there is very little temperature variation in the CMB (on the scale of μK); [1 mark]
- This indicates that there was very little temperature variation in the early universe too; [1 mark]

NOTE: when question asks to find the maximum rate of change of something, if a graph is given find the steepest gradient.

NOTE: in your answers, use same number of sf as values given in the question.